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L'EFFET PHOTOVOLTAIQUE SUR LES COUCHES MINCES DE TELLURURE DE PLOMB

ELENA ZAMFIR et CANDIDA OANCEA

Résumé

On étudie l'effet photovoltaïque des couches minces de PbTe deposées sous une incidence normale et oblique ($\theta = 0$), en visible et infrarouge ($\lambda = 10,6\mu m$). On constate l'apparition de l'effet seulement quand les couches sont obliquement déposées, la phototension étant dependente de l'épaisseur du film de PbTe.

INTRODUCTION

Dans des travaux anterieures (1), (2) ou a montré que les couches minces de tellure deposées sous une incidence oblique présentent un effet photovoltaïque, due en principal à l'orientation des cristallites. De même, on a montré (3) qu'au contact de la couche mince de tellure et celle de plomb apparait un effet photovoltaïque dont la valeur dépend entre autres des conditions de formation du film. Ainsi la phototension est plus grande dans le cas où le film de tellure a été obliquement déposé, en rapport avec le cas où il a été normalement déposé. L'étude de l'évolution en temp de l'effet au contact Te-Pb a mis en évidence un processus de diffusion entre ces deux éléments qui peuvent former le composé PbTe.

Dans ce travail nous avons étudié les propriétés photovoltaïques des couches minces de tellurure de plomb, déposées sous une incidence oblique et normale.

La structure et les propriétés électriques et optiques des couches minces de la tellurure de plomb ont été étudiées (4), (5), (6), à cause de l'intérêt présenté par ce composé semiconducteur pour la microélectronique, les processus de détection etc.

LA PARTIE EXPERIMENTALE

Les couches minces de tellurure de plomb ont été obtenues par évaporation thermique dans un vide de 10^{-5} torr sur des supports de verre et NaCl. Les cellules, de dimensions (40×7) mm^2 ont été préparées en déposant premièrement les électrodes de plomb et puis la couche de tellurure de plomb sous une incidence normale et oblique (0 = 60° entre la normale du support et la direction du faisceau moléculaire).

Les épaisseurs des couches, entre 0,05 — 8 μm, ont été determinées par une methode interférométrique.

L'effet photovoltaïque a été mesuré au long de l'épreuve à l'aide d'une installation composée par: une ampoule de projection de 750 W comme source de lumière, un modulateur mécanique et un système de lentilles, qui permetent la focalisation du faisceau et l'irradiation de l'épreuve par des impulsions.

L'épreuve à l'aide d'un dispositif se deplace en face d'un spot lumineux. La phototension obtenue a été visualisée à l'aide d'un duoscope après une préalable amplification.

LES RÉSULTATS EXPERIMENTAUX ET L'INTERPRÉTATION

L'étude de l'effet photovoltaïque a été fait par l'irradiation de l'épruve en impulsions, point par point.

Dans la figure 1 est représentée la phototension en fonction de la position du spot au long de l'épreuve dans laquelle la tellurure de plomb a été

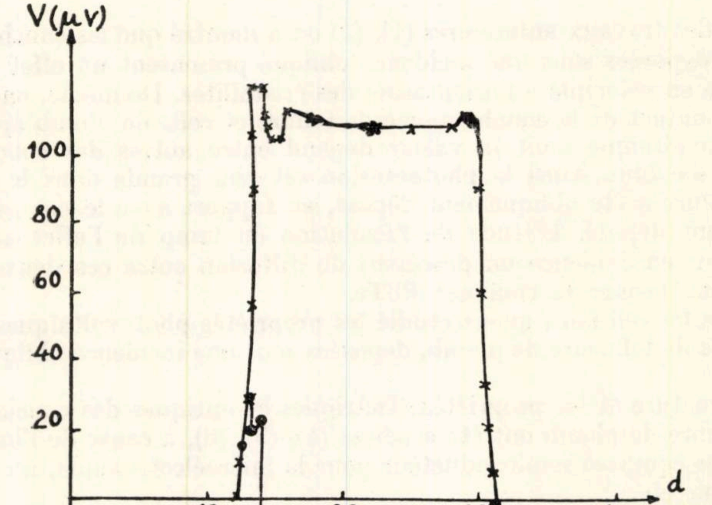

Fig. 1 — Les composantes: rapide (x) et lente (0) de la phototension en fonction de la position du spot lumineux au long de l'epreuve dans laquelle la tellurure de plomb a ete deposée sous une insidence de 60°, al'irradiation frontale.

déposée sous une incidence de 60°. L'épreuve a été irradiée sur la surface qui est en contact avec l'air. On constate que l'effet est formé par deux composantes: une qui est rapide et l'autre lente, qui ont été mésurées à osciloscope.

On constate l'éxistence d'un effet photovoltaïque sur toute la région comprise entre les électrodes à cause de la tellurure de plomb obliquement déposée. Pour cette épreuve, avec une épaisseur moyenne de 5800 Å et un gradient d'épaisseur de 188 Å/mm, la valeur de la phototension est pratiquement constante dans la région entre les électrodes. En même temps aux contacts de la couche de PbTe avec le Pb, on remarque une valeur un peu plus grande, due au contact métal-semiconducteur.

Dans la figure 2 est représentée la phototension au long de l'épreuve dans le cas où la tellurure de plomb a été normalement déposée sur les électrodes de plomb. On constate que dans la région comprise entre les électrodes l'effet photovoltaïque est pratiquement nul. Au contraire, on observe un effet photovoltaïque cu contact PbTe-métal avec une valeur plus petite que celle obtenue dans le cas d'une déposition oblique de la tellurure de plomb. L'épaisseur du film est la même avec l'épaisseur moyenne du film, dont l'effet photovoltaïque a été représenté fans la figure 1.

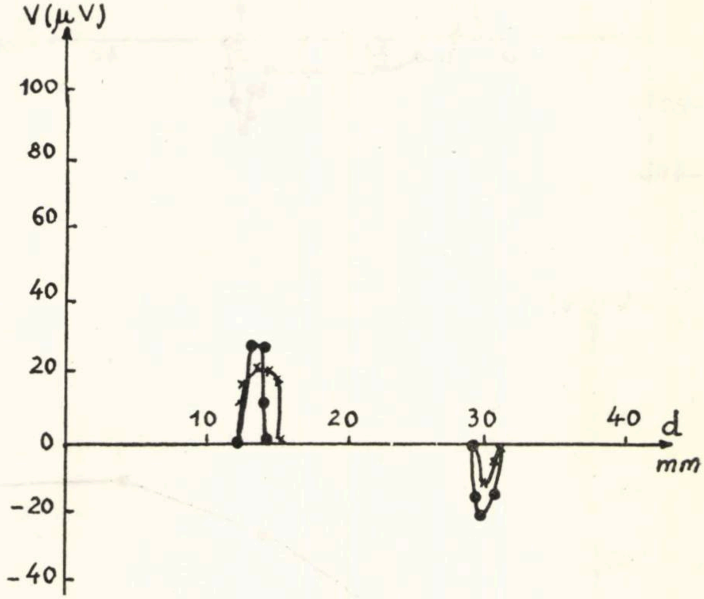

Fig. 2 — La phototension au long de l'épreuve avec la PbTe normalement déposée.

À l'irradiation de l'épruve à travers le support, on constate que la phototension garde la même polarité, mais l'intensité des deux composantes est plus petite en rapport avec le cas précedent. Dans la figure 3 est représentée la phototension au long de l'épreuve à l'irradiation à travers le support, pour l'épreuve de la figure 1. L'abaissement des valeurs des composantes de la phototension due à l'absorption de la lumière par le support a été seulement 8%

L'étude de l'effet photovoltaïque suivie sur un set d'épreuves de PbTe obliquement déposées, avec differents épaisseurs, obtenues dans les mêmes conditions de déposition, montre que la valeur de la phototension croît en même temps avec l'épaisseur du film, montrant une tendance de saturation. Dans la figure 4 est représentée la variation de la phototension avec l'épaisseur pour cinq épreuves. Les valeurs de la phototension et celles des épaiseurs ont été mesurées dans les points situées au milieu des épreuves.

Dans le cas d'une déposition oblique il y a un gradient d'épaisseur de la couche au long de l'épreuve, qui conduit a une variation de la phototension au long de l'épreuve pour des gradients d'épaisseur de 800—1200 Å/mm.

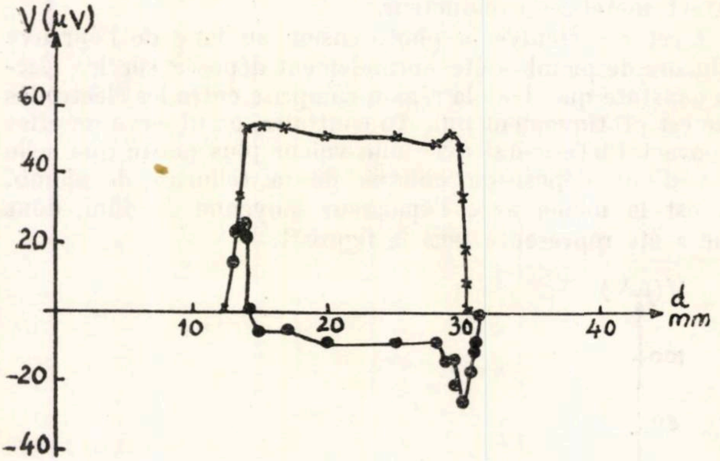

Fig. 3. — **La phototension au long de l'épreuve à l'irradiation à travers le support, pour le film de la figure 1.**

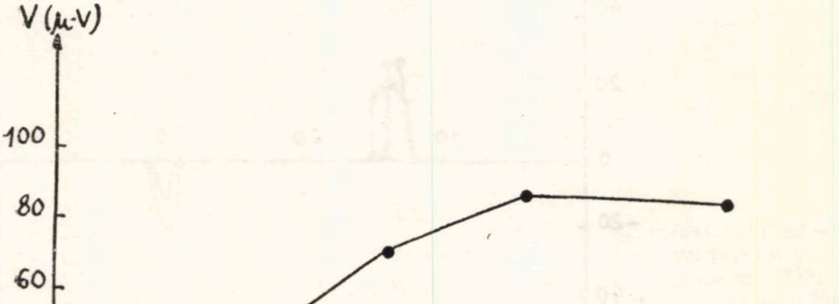

Fig. 4 — **La phototension en fonction de l'épaisseur du film**

L'effet photovoltaïque des couches minces de PbTe a été étudié aussi dans l'infrarouge, en utilisant la radiation = 10,6μm, donnée par un laser avec CO_2. Dans ce cas on a utilisé des cellules de dimensions 20×30 mm^2, dans lesquelles la tellurure de plomb a été déposée sous un angle d'incidence de 60° sur des supports de NaCl, clivée ou polie. On a obtenu des phototensions avec des valeurs comprises entre 0,4—45 mV, en fonction de l'épaisseur de la couche et la puissance du laser. La responsivité maxime de 5 mV/W a été obtenue pour une couche de 1,77μ m à l'irradiation avec un faisceau laser d'une puissance de 10 W.

L'effet photovoltaïque dans ce domaine spectrale peut être utilisé dans la réalisation d'un détecteur dans l'infrarouge qui peut travailler à la température ambiante (7)

L'apparition de l'effet photovoltaïque dans le visible et l'infrarouge sur les couches minces semiconducteur de PbTe peut être atribuée en principal à l'orientation des cristallites dans la direction de l'évaporateur. Le phénomène est similaire avec celui produit sur les couches de Te, Ge, Si et des differents composée. À cause d'un phénomène d'orientation des cristallites du film, apparait une composante tangentielle des phototensions engendrées dans les cristallites, qui s'aditionnent au long de la direction de la déposition qui determinent l'apparition de la phototension aux extrémités de la cellule. Dans le cas de la déposition normale la somme des composantes tangentielles est nule, comme dans le cas des couches minces de tellure (2)

D'après ce modèle la valeur de la phototension croît avec la dimension de la cristallite, ce qui explique la variation de la phototension avec l'épaisseur. Les variations qui apparaissent à l'irradiation frontale ou à travers le support s'explique par la variation des vitesses de recombinaisons aux deux surfaces de la couche (8) et à l'absorption de la radiation par le support.

En conclusion dans ce travail on a mis en évidence un effet photovoltaïque sur les couches minces de PbTe obtenues par une incidence oblique, dans le visible et l'infrarouge et la dépendance du photoeffet de l'épaisseur de la couche et la modalité de la déposition.

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THE SURFACE TENSION OF POLYDISPERSE POLYMER SOLUTIONS

RODICA MOLDOVAN

Synopsis

The lattice-theory equations for the surface tension of polymer solutions based on the parallel-layer model have been extended to a polydisperse polymer with known number-average molecular weight ($\overline{M}_n$). The surface tension is expressed as a function of the total polymer volume fraction. The presence in these equations of the number-average molecular weight in the surface monolayer, $\overline{M}_n^m$ suggests that the equations can be employed for preferential adsorption studies of the polymer fractions.

§ 1. INTRODUCTION

The lattice-theory equations for the surface tension on the base of the parallel-layer model for the surface of monomer + r-mer mixtures, have been first calculated by I. Prigogine and J. Maréchal.[1]. These equations have been applied by G. L. Gaines, Jr.[2,3] to the study of monodisperse and bidisperse polymer solutions in inert solvents and checked by experimental measurements.

The purpose of this paper is to extend the Prigogine and Maréchal equations for surface tension to polydisperse polymer solutions in inert solvents and to give the equations in a form suitable for experimental measurements.

The surface tension is calculated at thermodynamical equilibrium. The monolayer model and the equating of chemical potentials of the solution components in the bulk and surface phases are employed.

We denote by μ_j the chemical potential of the j component of the solution, by F—the free energy and by G—the free enthalpy. Quantities associated with the monomolecular surface layer are denoted by a superscript m. The chemical potentials are given by the wellknown equations:

$$\mu_j = \left(\frac{\partial F}{\partial n_j}\right)_{TVn_{i \neq j}} = \left(\frac{\partial G}{\partial n_j}\right)_{Tpn_{i \neq j}} \tag{1}$$

and

$$\mu_j^m = \left(\frac{\partial F^m}{\partial n^m}\right)_{TVn_j n_{i \neq j}} - \gamma a_j \tag{2}$$

where pv^m is neglected.

In these equations n_j are the number of moles of j, T is the absolute temperature, V is the volume, p is the pressure and γ_j is the surface tension of the solution. The quantity a is the partial molar area of the component j[4].

In chemical potential calculation we limited ourselves to the simplest model, originally given by Flory[5], in which the liquid is represented by a regular crystalline lattice. The monomolecular and paralel-layer model have been employed in surface phase free energy calculation.

§ 2. THERMODYNAMICAL FUNCTIONS IN THE BULK PHASE

a. The entropy of the solution

The entropy of mixing have been calculated in the Flory's approximation, representing the liquid by a regular lattice in which each solvent molecule occupies one cell and each polymer molecule occupies j celle's where:

$$j = \frac{V_{P_k}}{V_s} \tag{3}$$

is the polymer molecule volume, V_{P_k}, divided by the solvent molecule volume, V_s. The distribution of the molecular weights in a polymer sample make j vary over a certain range:

$$j = \frac{1}{\delta} K_0, \quad \frac{1}{\delta} (K_0 + 1), \ldots \frac{1}{\delta} (K_0 + \Delta) \tag{4}$$

because j is proportional to

$$\frac{V_{P_1}}{V_s} = \frac{1}{\delta} \tag{5}$$

where V_{P_1} is the monomer volume. Quantities K_0 and $K_0 + \Delta$ are the minimum and maximum degree of polymerisation of the polymer molecules in the sample, respectively. We assume that the molecules are randomly distributed in the Flory's lattice.

We shall make the following notations: N_i — the number of molecules occuping i cells ($i = 1$ for the solvent and $i = j$ for the polymer) N — the total number of cells of the Flory's lattice, z — the number of nearest neighbours in the liquid lattice and W — the number of new configurations apparing after mixing. According to the fundamental Boltzmann equation:

$$S_{conf} = k \ \ln \ W \tag{6}$$

(k is the Boltzman constant) and calculating W in the known manner[6] we obtain:

$$-\frac{S_{conf}}{k} = \sum_i N_i \ln \frac{N}{N_i} - (\ln z - 1) \sum_i (i - 1) \ N_i + \ln z \sum_j N_j \tag{7}$$

To find the entropy of mixing, S_{mix}, we must substract the configurational entropies of the i pure components obtained by particularizing the equation (7) from the configurational entropy (7). In this manner we find:

$$S_{mix} = -R \sum_i n_i \ln \varphi_i \tag{8}$$

where we introduced the gaz constant, R, the number of moles, n_i, and the volume fraction of the i component in solution:

$$\varphi_i = \frac{i N_i}{N} \tag{9}$$

The formula (8) is the generalization of the Flory equation for a solution of $(\Delta + 1)$ r-mers.

The entropy of the solution may be put in the form:

$$S = \sum_i n_i s_i^0 - R \sum_i n_i \ln \varphi_i \tag{10}$$

where s_i^0 is the molar entropy of the pure i component.

b. The configurational energy

The configurational energy of the solution have been calculated like the sum of the energies of interaction between pairs of nearest neighbours, by supposing that the interaction energy between a polymer segment and each of its neighbours is not influence by the degree of polymerisation of the polymer molecule to which this polymer segment belongs. We suppose weak interactions between the polymer and solvent molecules, as it have already been mentioned, that is:

$$E_{conf} \leqslant TS_{conf} \tag{11}$$

In this manner we can neglect the influence of the difference between the polymer molecule division in cells of Flory's lattice and the division in monomers on the configurational energy of the system. The effect of this approximation on the free energy is small, taking into account the relation (11) and the fact that δ is nearly equal to 1.

The configurational energy calculation follows closely the way we performed it in a previous paper[6] and we find:

$$E_{conf} = n_s^0 e_s^0 + \sum_j n_{P_j}^0 e_{P_j}^0 + \alpha n_s \varphi_P \tag{12}$$

where we denote by subscript s and P the quantities associated with the solvent ($i = 1$) and the polymer, respectively. φ_P represents the total volume fraction of the polymer in the bulk phase:

$$\varphi_P = \sum_j \varphi_{P_j} \tag{13}$$

and α is the parameter in the theory of regular solutions. e_s^0 and $e_{P_j}^0$ are the molar energies of the pure solvent and pure j polymer, respectively.

c. The free enthalpy and the chemical potentials

The free enthalpy, $G = E - TS + pV$, for a homogenous system composed of n_s moles of solvent and of n_{P_j} moles of j polymer, using the expression (10) for the entropy of the solution and the relation (12) for the configurationl energy, is:

$$G = n_s(e_s^0 - Ts_s^0 + pv_s^0) + \sum_i n_{P_j}(e_{P_j}^0 - Ts_{P_j}^0 + pv_{P_j}^0) + \alpha n_s \varphi_P + \\ + RT\left[n_s \ln \varphi_s + \sum_j n_{P_j} \ln \varphi_{P_j}\right] \tag{14}$$

The chemical potentials of the components of the solution, obtained by applying relation (1), are:

$$\mu_s = \mu_s^0 + RT \ln \varphi_s + RT\left[\varphi_P - \sum_j \frac{1}{j} \varphi_{P_j}\right] + \alpha(\varphi_P)^2 \tag{15}$$

and

$$\mu_{P_j} = \mu_{P_j}^0 + RT \ln \varphi_{P_j} + RT\left[1 - j\,\varphi_s - j\sum_j \frac{1}{j}\,\varphi_{P_j}\right] + \alpha j(\varphi_s)^2 \tag{16}$$

where the range of j is given by (4) and we denote by μ_s^0 and $\mu_{P_j}^0$ the standard chemical potentials of the solvent and pure j polymer molecules, respectively.

§ 3. THERMODYNAMICAL FUNCTİONS IN SURFACĖ PHASĖ

a. Entropy of the surface phase

The entropy of the surface layer is given by a relation analogous to (10) concerning the surface phase composition, because for the parallel-layer model the effect of the surface on the configurational term is neglected:

$$S^m = \sum_i n_i^m s_i^{m0} - R \sum_i n_i^m \ln \varphi_i^m \tag{17}$$

We assume that the molecules are randomly distributed in the surface monolayer.

b. The configurational energy

The calculation of the monolayer configurational energy have been performed by using also the parallel-layer model. We consider the liquid as composed of a series of monomolecular layers and we separate a system with k monomolecular layers plus the surface layer. By applying the Defay and Prigogine's method[4] as in a preceding paper[6], we substract from the configurational energy of this system, those of the k monomolecular layers from the bulk phase. By this method we obtain:

$$E^{m}_{conf} = n^{m}_{s} e^{m0}_{s} + \sum_{j} n^{m}_{P_j} e^{m0}_{P_j} + \frac{N^{m}}{N_{Av}} \mathrm{l}\, \alpha \varphi^{m}_{s} \varphi^{m}_{P} + \frac{N^{m}}{N_{Av}} m\alpha (\varphi^{m}_{s} \varphi_{P} + \varphi_{s} \varphi^{m}_{P} - \varphi_{s} \varphi_{P}) \tag{18}$$

Here e^{m0}_{s} and $e^{m0}_{P_j}$ are the molar energies of pure s and pure P_j in surface phase, N^{m} is the number of cells in a monomolecular layer and N_{Av} is the Avogadro number. Quantities l and m are the fraction of nearest neighbours which occupy the same and an adjacent lattice plane, respectively.[4]

c. Free energy and chemical potentials in the surface phase

The free energy in the surface monolayer, $F^{m} = E^{m} - TS^{m}$, using the equations (17) and (18), becomes:

$$\begin{aligned} F^{m} = {} & n^{m}_{s}(e^{m0}_{s} - Ts^{m0}_{s}) + \sum_{j} n^{m}_{P_j}(e^{m0}_{P_j} - Ts^{m0}_{P_j}) + RT(n^{m}_{s} \ln \varphi^{m}_{s} + \sum_{j} n^{m}_{P_j} \varphi^{m}_{P_j}) + \\ & + \mathrm{l}\alpha n^{m}_{s} \varphi^{m}_{P} + m\alpha(n^{m}_{s}\varphi_{P} + n^{m}_{P}\varphi_{s}) - \Big(n^{m}_{s} + \sum_{j} j n^{m}_{P_j}\Big) m\alpha \varphi_{s} \varphi_{P} \end{aligned} \tag{19}$$

The chemical potentials, obtained by differentiation of relation (19), in accordance with (2) and employing the standard chemical potentials of the pure components in the surface monolayer, are:

$$\mu^{m}_{s} = \mu^{m0}_{s} + RT\Big[\ln \varphi^{m}_{s} + \varphi^{m}_{P} \sum_{j} \frac{1}{j} \varphi^{m}_{P_j}\Big] + l\alpha(\varphi^{m}_{P})^{2} + m\alpha\varphi^{2}_{P} - (\gamma - \gamma_{s})\, a_{s} \tag{20}$$

and

$$\begin{aligned} \mu^{m}_{P_j} = {} & \mu^{m0}_{P_j} + RT\Big[\ln \varphi^{m}_{P_j} + 1 - j\varphi^{m}_{s} \cdot \quad j \sum_{j} \frac{1}{j} \varphi^{m}_{P_j}\Big] + l\alpha j(\varphi^{m}_{s})^{2} + \\ & + m\alpha j(\varphi^{2}_{P})^{2} - (\gamma - \gamma_{s})\, a_{s} \end{aligned} \tag{21}$$

In these relations j varies according to (4) and γ_{k}, γ_{j} are the solvent and pure j polymer surface tensions, respectively.

§ 4. THE SURFACE TENSION OF THE SOLUTION

The expressions of the surface tension are obtained by equating the chemical potentials of the same component in the bulk [eqs (15) and (16)] and the surface [eqs (20) and (21)] phases, and taking into account the equalities between some standard chemical potentials previously discussed.[6] Then, because:

$$a_{P_j} = ja \tag{22}$$

we find:

$$\gamma = \gamma_{P_j} + \frac{RT}{a_s}\left[\ln\frac{\varphi_s^m}{\varphi_s} - (\varphi_s^m - \varphi_s) - \sum_j \frac{1}{j}(\varphi_{P_j}^m - \varphi_{P_j})\right] + \\ + \frac{\alpha l}{a_s}(\varphi_P^m)^2 + \frac{\alpha(m-1)}{a_s}\varphi_P^2 \tag{23}$$

and

$$\gamma = \gamma_{P_j} + \frac{RT}{ja_s}\left[\ln\frac{\varphi_{P_j}^m}{\varphi_{P_j}} - j(\varphi_s^m - \varphi_s) - j\sum_j \frac{1}{j}(\varphi_{P_j}^m - \varphi_{P_j})\right] + \\ + \frac{\alpha l}{a_s}(\varphi_s^m)^2 + \frac{\alpha(m-1)}{a_s}\varphi_s^2 \tag{24}$$

Because, generally, $\varphi_{P_j} \leqslant 1$ and $j \gg 1$, the logarithm in (24) can be expended, that is:

$$\frac{1}{j}\ln\varphi_{P_j} = \ln[1 - (1 - \varphi_{P_j})]^{1/j} \simeq -\frac{1}{j}(1 - \varphi_{P_j}) \tag{25}$$

and the equations (24) become:

$$\gamma = \gamma_{P_j} + \frac{RT}{a_s}\left[\frac{1}{j}(\varphi_{P_j}^m - \varphi_{P_j}) - (\varphi_s^m - \varphi_s) - \sum_j \frac{1}{j}(\varphi_{P_j}^m - \varphi_{P_j})\right] + \\ + \frac{\alpha l}{a_s}(\varphi_s^m)^2 + \frac{\alpha(m-1)}{a_s}\varphi_s^2 \tag{26}$$

The expressions of the surface tension in equation (26) are not suitable for practical purposes because both γ_{P_j} and φ_{P_j} are very difficult to be measured experimentaly. We know the total volume-fraction of the polymer from the solution preparation which is very nearly equal to φ_P from (13). We look for expressions of the surface tension of the solution as a function of φ_P. For this purpose we add the $(\Delta + 1)$ equations (26) for the polymer and we find:

$$\gamma = \frac{1}{\Delta+1}\sum_j \gamma_{P_j} - \frac{RT}{a_s}\left[\sum_j \frac{1}{j}(\varphi_{P_j}^m - \varphi_{P_j}) + (\varphi_s^m - \varphi_s)\right] + \\ + \frac{RT}{(\Delta+1)\,a_s}\sum_j \frac{1}{j}(\varphi_{P_j}^m - \varphi_{P_j}) + \frac{\alpha l}{a_s}(\varphi_s^m) + \frac{\alpha(m-1)}{a}(\varphi_s)^2 \tag{27}$$

In accordance to relation (9) we have:

$$\sum_j \frac{1}{j}\varphi_{P_j} = \frac{\sum\limits_j N_{P_j}}{N} = \frac{N'}{N} \tag{28}$$

where N' is the total number of polymer molecules and $N = N' + N_s$. But

$$\varphi_P = \frac{\sum\limits_j jN_{P_j}}{N} \tag{29}$$

so we must express N/N' as a function of φ_P.

We write the number-average molecular weight expression for the polymer, $\overline{M}_n$:

$$\overline{M}_n = \frac{m_0 \sum_k k N_{P_k}}{N'} \tag{30}$$

where m_0 is the molecular weight of the monomer and $k = \frac{V_{P_k}}{V_{P_1}}$ is the degree of polymerisation of polymer molecules.
Then:

$$\frac{\varphi_P}{\overline{M}_n} = \frac{1}{m_0} \frac{N'}{N} \frac{1}{\delta} \tag{31}$$

because the ratio of the two sums in (29) and (30) is equal to $1/\delta$ according to (5).
Therefore:

$$\sum_j \frac{1}{j} \varphi_{P_j} = m_0 \delta \frac{\varphi_P}{\overline{M}_n} \tag{32}$$

The molecular weight distribution in the surface and in the bulk phases must be different and, introducing the number-average molecular weight in the monolayer, $\overline{M}_n^m$, we have:

$$\sum_j \frac{1}{j} \varphi_{P_j}^m = m_0 \delta \frac{\varphi_P^m}{\overline{M}_n^m} \tag{33}$$

Making use of eqs (32) and (33) and introducing the surface tension of the pure polydisperse polymer, γ_P, equation (27) becomes:

$$\gamma = \gamma_P + \frac{RT}{a_s}\left[m_0\delta\left(\frac{\varphi_s^m}{\overline{M}_n^m} - \frac{\varphi_s}{\overline{M}_n}\right) - (\varphi_s^m - \varphi_s)\right] - \frac{RT}{(\Delta+1)\,a_s} m_0\delta\left(\frac{\varphi_s^m}{\overline{M}_n^m} - \frac{\varphi_s}{\overline{M}_n}\right) + \\ + \frac{\alpha l}{a_s}(\varphi_s^m)^2 + \frac{\alpha(m-1)}{a_s}(\varphi_s)^2 \tag{34}$$

Folowing the same method of calculation, thr equation (23) becomes:

$$\gamma = \gamma_s + \frac{RT}{a_s}\left[\ln\frac{\varphi_s^m}{\varphi_s} - (\varphi_s^m - \varphi_s) - m_0\delta\left(\frac{\varphi_P^m}{\overline{M}_n^m} - \frac{\varphi_P}{\overline{M}_n}\right)\right] + \frac{\alpha l}{a_s}(\varphi_P^m)^2 + \\ + \frac{\alpha(m-1)}{a_s}(\varphi_P)^2 \tag{35}$$

5. DISCUSSION

Equations (34), (35) contain the unknown quantities φ_s^m (or φ_P^m) and $\overline{M}_n^m$ while the quantities φ_s(or φ_P), γ_s, γ_P, a_s, m_0, $\overline{M}_n$, δ, α, 1, m, Δ are known either from the solution preparation, or from the specific features of the polymer. Particularly, it is known that Δ can be expressed with respect to $\overline{M}_w/\overline{M}_n$

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where $\overline{M}_w$ is the weight-average molecular weight of the polymer. Only by making assumption that

$$\overline{M}_n^m \cong \overline{M}_n \qquad (36)$$

as a first approximation, equations (34) and (35) can be used to calculate the surface tension of the solution, γ. But, in our opinion, it would be more interesting to use equations (34), (35) for the study of preferential adsorption of the different polymer fractions at different polymer concentrations. by calculating $\overline{M}_n^m$ from $\gamma = \gamma(\varphi_P)$ experimentaly known. $\overline{M}_n^m$ values greater or smaller than $\overline{M}_n$, respectively, will reveal preferential adsorbtion of polymers with greater or smaller molecular weight. We stress that one cannot say anything *a priori* about the preferential adsorbtion because the polymer surface tension dependence of the molecular weight is not fully understood.

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ASUPRA FLEXIBILITĂŢII DE LANŢ ŞI DENSITĂŢII DE ÎMPACHETARE ALE REŢELELOR POLIMERICE

ION C. PETREA, LIVIA IONESCU

ABSTRACT

A theree—dimensional polymer with a small reticulation degree and a rather high packing coefficient offres essential characteristics for systematic elucidation of its various structural — morphological interdependences.

The constuction of a suitable model allows to esiablish a series of relations illustrating the interconnections of the molecular mass, polymmerization degree, cross-link density, length of the elementary chain, packing density.

INTRODUCERE

În structura unui polimer tridimensional pot fi puse în evidență prin metode adecvate de analiză structurală, domenii caracterizate fie prin predominanța unor laticii, fie orientarea preferențială a macrolanțurilor relativ împachetate. Aceste domenii alternează, intervenind unele sau altele cu o pondere mai mare sau mai mică, imprimînd corespunzător caracterul de polimer linear sau de polimer reticulat sistemului macromolecular în care pachetele de lanțuri macromoleculare au orientări determinate de pre-istoria producerii materialului polimeric, adică de procedeul tehnologic de obținere a polimerului. [1—5].

CORELAREA FLEXIBILITĂŢII ŞI RETICULĂRII MACROLANŢURILOR

Proprietățile macrofizice ale unui polimer cu structură de lanțuri orientate depind de constituția chimică a acestora, care este primordial responsabilă de flexibilitatea și împachetarea macromoleculelor. Orientarea macrolanțurilor și a structurilor supermoleculare ca și dimensiunile acestora influențează mai ales proprietățile mecanice, dependente totodată și de densitatea de reticulare:

$$\mathrm{d}_r = q \cdot \frac{m_0}{\rho} = \frac{\nu}{V} = \frac{\rho}{M_{ee}} \qquad (1)$$

unde, proporția elementelor monomerice din lanț ce revin unei punți transversale, se notează cu d_r. În definirea gradului de reticulare a unui polimer tridimensional se observă din relația (1) că intervin atît masa m_0 a monomerului, cît și densitatea ρ a polimerului; ν — numărul de moli.

Un parametru cu semnificație analogă mărimii d_r ca măsură a reticulării este q, adică numărul lanțurilor elementare (Fig. 1) din unitatea de volum

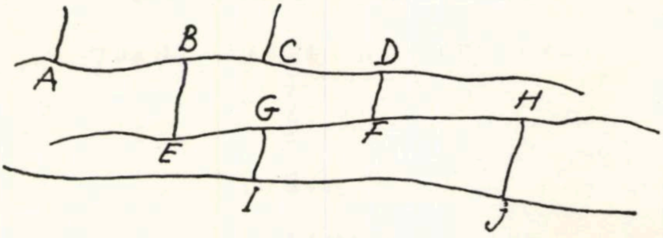

Figura 1.

În figura 1, AB, BC, CD etc. sînt lanțuri elementare; iar BE, DF, GI, etc. sînt punți transversale.

R. Famechon [6], ținînd seama de caracterul statistic al gradului de polimerizare, precum și de semnificația densității de reticulare d stabilește o legătură între aceste mărimi introducînd o mărime p_0 — numită coeficient de pontaj, care ilustrează numărul punților aferente unei molecule:

$$p_0 = d_r \langle P \rangle \qquad (2)$$

Autorii prezentei lucrări au stabilit într-un studiu anterior [7] și alte relații între parametrii caracteristici structurilor cristaline și reticulate concomitent, cum sînt, de exemplu, gradul de cristalizare g_c, masa moleculară medie $\langle M \rangle$ coeficientul de pontaj p_0, așa cum se observă în relația:

$$g_c = \frac{q}{p} \cdot \frac{\langle M \rangle}{m'} \cdot \left(V - \frac{m''}{\rho'} \right) \qquad (3)$$

unde m' este masa sistemului cristalizabil la începutul procesului de cristalizare, ρ' — densitatea fazei amorfe, m'' — masa amorfă, iar V — volumul polimerului.

Este știut că pentru a obține fibre elastice și superelastice rezistente și durabile, tehnologia modernă urmărește producerea unor polimeri care să se caracterizeze prin energie de coeziune înaltă. Asemenea polimeri sînt aceia care au o structură de macrolanțuri orientate, dens împachetate.

Cu alte cuvinte, se are în vedere faptul că energia interacțiunilor intermoleculare descrește rapid cu creșterea distanțelor dintre molecule. Macrolanțurile rigide nu se împachetează compact așa cum se întîmplă cu cele foarte flexibile din polimerii preferați pentru producerea fibrelor și firelor sintetice supraelastice. Din această cauză, polipropilena, de exemplu, avînd o structură de macrolanțuri flexibile, și împachetate, constituie o materie primă deosebit de importantă pentru fabricarea unor asemenea fire sau fibre.

Orientarea pachetelor de macrolanțuri în structurile polimere conferă acestora o rezistență mecanică comparabilă cu a oțelurilor superioare (cca 120 Kgf/mm^2). Astfel, rezistența mecanică a firelor de in se datorește prin excelenţă orientării înalte a macromoleculelor care îi alcătuiesc structura. Macromoleculele altor structuri celulozice sînt mai puţin orientate şi împachetate ca atare, şi astfel, rezistenţa lor mecanică este mai mică.

Orientarea şi împachetarea macrolanţurilor în poliamide şi poliesteri determină tăria fibrelor confecţionate din acestea, superioare firelor de lînă şi de mătase naturală. Fibra de poliacrilonitril — orlon — este din aceleaşi considerente mai rezistentă decît lîna naturală.

FLEXIBILITATEA ŞI ÎMPACHETAREA MOLECULELOR POLIMERE

Este interesant de remarcat că fibrele elastice nu se obţin din polimeri înalt-elastici. Polimerii solizi care conţin grupe capabile de puternice interacţiuni intermacromoleculare sînt deasemenea utilizaţi în acest scop.

Efectul acestor interacţiuni ale lanţurilor adiacente poate fi asimilat efectului datorat punţilor transversale din sistemele macromoleculare reticulate [8]. Orientarea, împachetarea şi pseudoreticularea prin prezenţa acestor grupe atomice concurează atît la creşterea rezistenţei mecanice cît şi la rearanjarea lanţurilor în structurile supermoleculare. Masa moleculară M_{le} a lanţului elementar dă o măsură a densităţii de reticulare, indicînd totodată şi compacticitatea pachetului ca formaţiune structurală supermoleculară. În acest sens se propune să se definească coeficientul de compacticitate al pachetului de lanţuri moleculare prin raportul dintre masa moleculară a polimerului şi masa moleculară a lanţului elementar:

$$\frac{\langle M \rangle}{M_{le}} \equiv c_i \tag{4}$$

observînd că acest coeficient, c_i poate fi o expresie a densităţii de împachetare a macrolanţurilor în formaţiunile supermoleculare.

În concordanţă cu relaţiile (1), (2) şi (3), dacă se observă, de exemplu, că densitatea de reticulare depinde atît de masa specifică a polimerului, cît şi de masa moleculară a lanţului elementar, se poate recurge la o definiţie mai comodă pentru densitatea de împachetare, luînd în consideraţie masa $m = \rho V$ în locul mărimii M_{le} şi astfel, vom avea:

$$\frac{m}{M_{le}} = d_i \tag{5}$$

Experimental, metodologia curentă de investigare a structurilor macromoleculare, permite estimarea parametrului M_{le}, care oferă o posibilitate de corelare şi interpretare comodă a mecanismului flexibilităţii de lanţ, împachetării şi reticulării proprii unor clase de polimeri sintetici. Numărul legăturilor transversale formate poate fi estimat, de exemplu, prin cantitatea de hidrogen eliberată. În general, dacă numărul punţilor devine prea mare, M_{le} se micşorează şi densitatea de împachetare d_i creşte, afectînd pe de o parte flexibilitatea lanţurilor, iar pe de altă parte, macroscopic, înalta elasticitate, pendinte de rotaţiile interne ale unităţilor monomere din lanţurile sistemelor elastomere.

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CONTRIBUŢII PRIVIND ACŢIUNEA CÎMPULUI MAGNETIC ASUPRA SEPARĂRII UNOR MINERALE SULFUROASE

VASILE P. MIHU, IULIU POP

Synopsis

The magnetic field can influence upon the metal separation from the nonferrous minerals.

Experimental researches in labor and industrial phase showd the existence of an alternate nonperiodical function that characterises the connection between the magnetic field intensity and the metal extraction.

These functions are superimposing within same ranges for which positive controllabe results for one or several metals are obtained.

The results obtained under labor-phase allowed the realisation and introduction of an industrial device for magnetic treatment of mineral mixture-reactant solutions.

INTRODUCERE

În tehnologia preparării minereurilor prin flotaţie se separă mineralele fin măcinate creîndu-se condiţiile de hidrofobitate, sau hidrofilitate a acestora. Metodele clasice de utilizare a unor reactivi pot fi completate cu unele metode noi bazate pe acţiunea unor cîmpuri ultraacustice (1,2) sau magnetice. În cele ce urmează se aduc unele contribuţii privind acţiunea cîmpului magnetic asupra selectivităţii de separare a unor minerale sulfuroase. Pe plan mondial sînt cunoscute lucrările lui Minenko, Petrov, Mint (I) şi ale lui Klassen (4) prin care în diferite sisteme minerale-soluţie se constată efecte pozitive sau negative ale cîmpului magnetic asupra selectivităţii procesului de preparare. Lucrarea de faţă aplicată unor minereuri din bazinul minier Maramureş stabileşte experimental o funcţie ce leagă intensitatea cîmpului magnetic de extracţia de metale neferoase.

§ 1. CONDIŢII DE LUCRU

Încercările experimentale au fost efectuate, atît în fază de laborator cît şi în fază industrială, asupra unor minereuri conţinînd ca minerale utile, blendă, galenă, calcopirit, iar ca minerale balast, bioxidul de siliciu, feldspaţi etc. Mineralele în soluţia de reactivi (tulbureală) au fost trecute printr-o conductă aşezată între polii a cinci electromagneţi. Numărul de electromagneţi a fost stabilit luînd în considerare o eficienţă maximă a cîmpului magnetic la un debit dat al amestecului minerale-soluţie de reactivi şi de asemenea optimizarea între costul instalaţiei şi a efectului maxim al cîmpului magnetic. (5).

Instalaţia de magnetizare cu cîmp magnetic transversal realizată în laborator cît şi cea industrială realizată prin amplificarea la scară a instalaţiei de laborator au următoarele caracteristici principale:

— electromagneţi : 5
— intensitatea maximă a curentului : 20 Acc
— intensitatea maximă a cîmpului magnetic : 120 KAm^{-1}
(1500 Öe)

Instalaţia de magnetizare industrială a fost montată la Exploatarea Minieră Baia-Borşa, judeţul Maramureş, unde minereul provenit de la mina Burloaia este supus procesului de preparare în două linii tehnologice paralele şi identice. În acest fel s-a putut considera una dintre linii ca etalon a liniei de preparare, în care s-a introdus tratarea magnetică a amestecului minerale-soluţie de reactivi.

§ 2. REZULTATE EXPERIMENTALE

Rezultatele încercărilor experimentale efectuate în fază industrială sînt ilustrate în tabela nr.1

Variaţia intensităţii cîmpului magnetic între 0—90 KAm , arată că acesta are influenţă asupra extracţiei de metale pentru Cu, Pb, Zn. Cele mai bune rezultate s-au obţinut la intensitatea magnetică de 88 KAm cînd în concentratul colectiv de cupru şi plumb s-a obţinut extracţie de 94,7% Cu, 93,8% Pb şi 20,2% Zn, faţă de 89,5% Cu, 92,4% Pb şi 36,8% Zn obţinute în lipsa cîmpului magnetic. Este de menţionat că în concentratul de cupru şi plumb se tinde reducerea extracţiei de zinc deoarece mineralul de zinc este colectat separat de mineralele de cupru şi plumb.

Pentru a ilustra mai concludent măsurătorile efectuate, în fig. 1 sînt indicate graficele variaţiilor extracţiilor de Cu, Pb, Zn, în funcţie de intensitatea cîmpului magnetic.

Este de ramarcat că faţă de probele etalon ies în evidenţă variaţiile după o funcţie alternantă a extracţiilor de metale. Maximele funcţiilor ce caracterizează extracţiile se află situate după cum urmează:

— în cazul cuprului, unde se observă cele mai importante variaţii ale extracţiei în funcţie de intensitatea cîmpului magnetic, apar influenţe pozitive în domeniile 5—25 KAm^{-1} 35—75Am^{-1} şi peste 85 KAm^{-1}. În domeniile 25—35 KAm^{1}, se constată minime ale funcţiei extracţiei. În general magnetizarea amestecului minereu-soluţie de reactivi are un efect pozitiv;

— în cazul plumbului, funcţia extracţiei are salturi mai mici, efectele pozitive faţă de proba etalon observîndu-se în domeniile 5—35 KAm^{-1}, 55—

Tabela 1.

Determinarea intensităţii optime la flotaţia concentratului colectiv de Pb-Cu în fază industrială.

Produsul	Extracţie în greutate, %	Conţinut, %			Extracţie, %			H KAm^{-1}
		Cu	Pb	Zn	Cu	Pb	Zn	
Original calculat	100,0	0,87	1,62	3,31	100,0	100,0	100,0	Etalon
Concentrat Pb-Cu	34,0	2,30	4,42	3,6	89,5	92,3	36,8	
Steril	66,0	0,14	0,19	10,5	7,7	63,2	63,2	
Original calculat	100,0	0,84	1,61	3,25	100,0	100,0	100,0	8
Concentrat Pb-Cu	37,7	2,10	3,98	3,86	94,1	93,0	44,7	
Steril	62,3	0,08	0,18	2,88	5,9	7,0	55,3	
Original calculat	100,0	0,85	1,61	3,28	100,0	100,0	100,0	16
Concentrat Pb-Cu	36,8	2,16	4,06	4,00	93,7	93,1	44,8	
Steril	63,2	0,085	0,17	2,86	6,3	6,9	55,2	
Original calculat	100,0	0,83	1,64	3,29	100,0	100,0	100,0	24
Concentrat Pb-Cu	37,3	2,10	4,06	4,25	94,7	92,8	48,3	
Steril	62,7	0,07	0,18	2,70	5,3	7,2	51,7	
Original calculat	100,0	0,90	1,63	3,24	100,0	100,0	100,0	32
Concentrat Pb-Cu	39,2	2,04	3,90	4,05	88,2	93,3	48,8	
Steril	60,8	0,077	0,18	2,74	11,8	6,7	51,2	
Original-calculat	100,0	0,88	1,67	3,50	100,0	100,0	100,0	40
Concentrat Pb-Cu	37,2	2,23	3,98	4,28	94,2	92,0	45,5	
Steril	62,8	0,082	0,20	3,05	5,8	8,0	54,5	
Original calculat	100,0	0,86	1,67	3,42	100,0	100,0	100,0	48
Concentrat Pb-Cu	35,2	2,29	4,34	3,92	93,7	91,0	40,0	
Steril	64,8	0,083	0,23	3,16	6,3	9,0	60,0	
Original calculat	100,0	0,83	1,66	3,49	100,0	100,0	100,0	56
Concentrat Pb-Cu	34,2	2,29	4,53	4,25	94,1	41,5	41,5	
Steril	65,7	0,17	3,10	3,10	7,0	7,0	58,5	
Original calculat	100,0	0,34	1,68	3,44	100,0	100,0	100,0	64
Concentrat Pb-Cu	32,1	2,75	4,84	4,16	94,1	92,5	39,0	
Steril	67,9	0,08	0,18	3,10	5,9	7,5	61,0	
Original calculat	100,0	0,94	1,85	3,54	100,0	100,0	100,0	72
Concentrat Pb-Cu	39,5	2,28	4,37	4,04	94,0	93,1	45,2	
Steril	60,5	0,08	0,21	3,20	5,0	6,9	54,8	
Original calculat	100,0	0,85	1,64	3,44	100,0	100,0	100,0	80
Concentrat Pb-Cu	26,1	3,00	5,62	4,10	91,7	89,2	31,0	
Steril	73,9	0,09	0,24	3,20	8,3	11,8	69,0	
Original calculat	100,0	1,37	1,37	2,93	100,0	100,0	100,0	88
Concentrat Pb-Cu	27,1	4,81	6,48	2,20	94,7	93,8	20,2	
Steril	72,9	0,10	0,61	3,20	5,3	6,2	79,8	

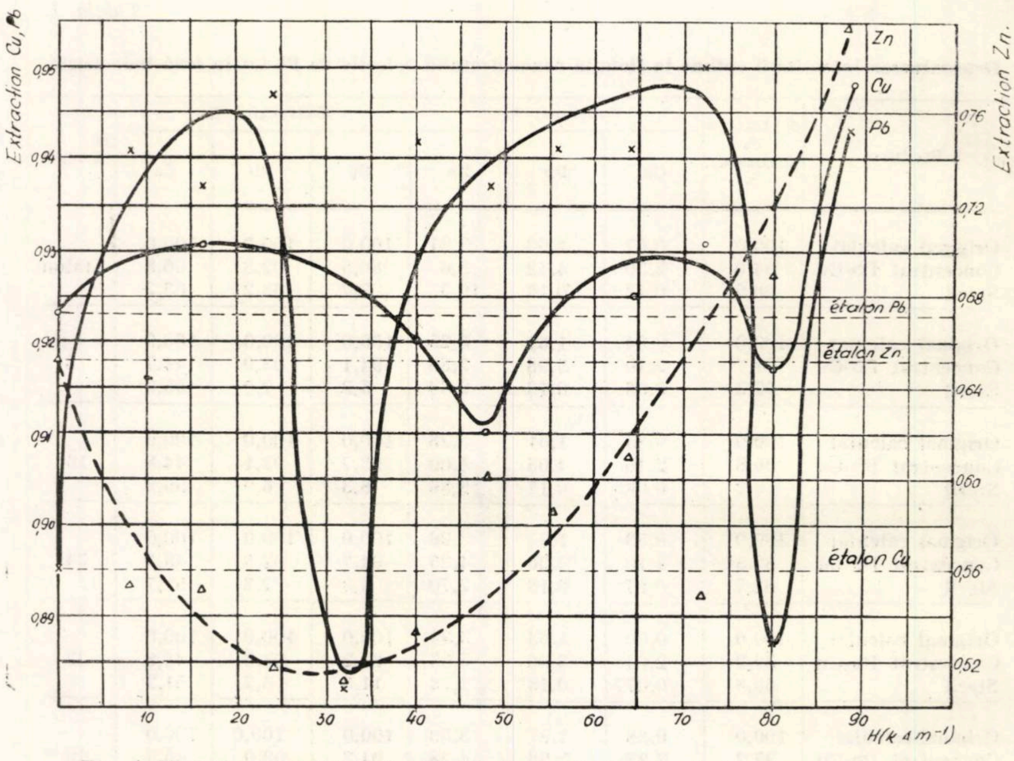

Fig. 1. Variaţia extracţiei de Cu, Pb, Zn în funcţie de intensitatea cîmpului magnetic.

75 KAm^{-1} şi peste 85 KAm^{-1} iar efectele negative fiind sesizate în domeniile 40—50 KAm ^{1} şi un domeniu strîns în jurul valorii 80 KAm ^{1}, aproape insesizabil. Este de remarcat că tratarea magnetică a amestecului mineral-soluţie de reactivi are o influenţă mai mică asupra extracţiei de plumb;

— în cazul zincului s-a trasat curba ce indică extracţia de zinc în ceea ce rămîne după flotarea plumbului şi cuprului, în cazul dat, sterilul. Această curbă este mai reprezentativă deoarece în procesul de preparare se urmăreşte separarea zincului de cupru care flotează împreună cu plumbul. Curba reprezentată pe grafic indică un efect negativ al magnetizării în domeniul 0—70 KAm ^{1}, urmată de un domeniu în care efectul tratării magnetice se manifestă în mod foarte pregnant.

Din studierea curbelor trasate în fig. 1 cît şi din tabela 1 a rezultat că tratarea magnetică trebuie realizată cu cîmpuri magnetice avînd intensitatea cuprinsă în domeniul 85—90 KAm^{-1}.

Aplicarea celor observate mai sus în fază industrială pe o perioadă îndelungată, de ordinul lunilor, a consemnat influenţa favorabilă a cîmpului magnetic de o o anumită intensitate asupra creşterii extracţiei de metale în flotarea minereurilor neferoase.

§ 4. CONCLUZII

1. Cîmpul magnetic transversal are o influenţă asupra extracţiei de metale din mineralele neferoase. Caracteristica cîmpul magnetic-extracţie de metale este o funcţie alternantă neperiodică.

2. Intensităţile optime ale cîmpului magnetic se intind pe anumite zone ale valorilor cîmpului magnetic şi sînt caracteristice fiecărui metal. Este posibil ca anumite zone ale valorii intensităţii cîmpului magnetic să coincidă, în acest caz efectul cîmpului magnetic se extinde favorabil pentru metalele respective.

3. Funcţiile alternante neperiodice ce caracterizează extracţiile de metal pot fi utilizate pentru îmbunătăţirea selectivităţii procesului de preparare, cînd se pot crea simultan zone de influenţă pozitive sau negative controlabile entru un singur metal sau mai multe metale.

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AN AUTOMATIC ADJUSTABLE THERMOSTAT FOR LOW TEMPERATURE*

G. G. BRĂTESCU and A. V. GROSSU

Summary

In this paper there is a description af an on-off thermostat for the domain of temperature of −60, +20°C, having an accurancy of 0,5°C. The working principle and the results obtained, regarding the maintaining of a constant temperature, and the consumption of solid CO_2 are discussed.

INTRODUCTION

There are many thermostats on the domain of temperature of −60, +20°C, which are based on different principles of maintaining a constant temperature, having different performances. They can be divided in proporţional thermostats and on-off thermostats. (I), (13), (14)

In the first kind of thermostats, the heat agent and the cold one which are in echilibrium connected with the maintaining of the constant temperature, are working in every moment, their action beeing a proportionally one. The accurancy of maintaining a constant temperature is better as 0,1°C on the choosed domain. The control element of temperature adjusting is usually a resistance, connected to a bridge of alternative current whose action is continuously controlling the effect of the cold and heat agent. These systems are sensitive at the rapid change of the external agents. (I), (24).

The on-off systems are based on the alternative action of the cold heat agents. (I). The accurancy of maintaining of a constant temperature is 0,1°C on the domain, the controll element is usually a contact thermometer. (Is) These systems are caracterized by a simple safe working, over a long time. They are in the same time insensitive at the rapid action of the external agents. (19), (24).

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THE DESCRIPTION AND THE WORKING SYSTEM OF THE THERMOSTAT

The system used is represented by a kind of an on-off thermostat It uses the forced circulation of the thermostated liquid. (I), (30). The heat agent is the action of the sorroundings and the cold agent is represented by the action of solid CO_2 + acetone (10), which is not entering in the circulation of the thermostated liquid. The cold agent is automatically controlled by a valve connected with a relay. This valve can be changed with a tap of an variable opening, whose position must be fixed for every point of temperature. The heat action of the sorroundings can be changed by the action of a heat resistance. This system is caracterized by a big consumption of solid CO_2.

The cold agent ensure a rapid coldness of the thermostated liquid because solid CO_2 have a high cold power of 150kcal/Kg. (10).

The thermostated liquid is represented by ethylic alcohol which has a low viscosity on this domain of temperature, beeing in the same time unharmful (7), (2).

The coldness of the thermostated liquid is achieved in the cold tank. It is built up by three coaxial cylinders, which are forming the tank with the mixture of solid CO_2 + acetone, the jacket for the circulation of the thermostated liquid, and the double walls for termic isolation. The cold tank has also a level for the following of the thermostated liquid inside the circulation jacket. The tank is built up by sheet of copper $\neq$ I.

The cold tank has the following form, described by Fig..1.

The valve with relay, has the aim of changing the direction of the thermostated liquid. It is built up by two cylinders of brass and two valves which are acting inside of these cylinders. The two valves are fixed by a changeable

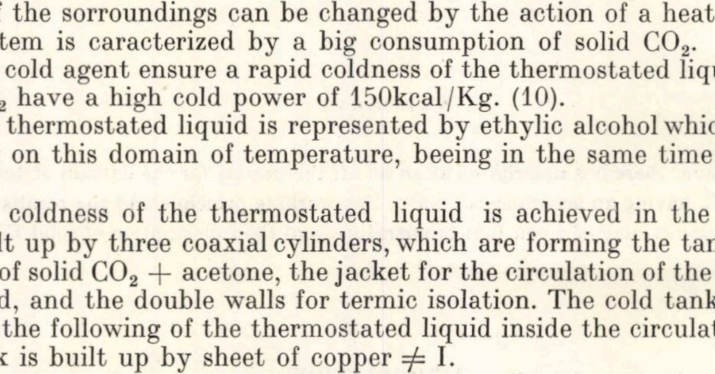

Fig. 1. The design of the cold tank

1. The level for the following of the liquid inside the circulation jacket
2. wall for thermic isolation
3. Jacket for the circulation of the thermostating liquid
4. tank containing the mixture of solid CO + acetone
5. entering connection
6. exit connection

distance of two cylinders of iron, which can move by the action of the induction field created by two spools inside the cylinders of brass. The spools are built up by 9000 helix of copper wire 0,22. The connection pieces with double thread, at the upper side of the brass cylinders built up by OL-50 are creating a strong induction field of the two spools. With the exception of the valves, connection pieces with double thread and the action cylinder, the other pieces are built up by brass. The connections of the relay with the other tanks is made by four connection pieces which can be changed each others.

The relay is represented by Fig. 2.

We specify that the relay has not thermic isolation.

The closed consumer, is built up which the aim of obtaining of a large thermostated bath, of a good thermic isolation. Its volum is of I l, and is built up by two cylinders of copper sheet ≠ 1, which are forming inside the thermostatic bath and outside the isolation walls. The closed consumer has at the upper side a wall of plexiglass, which is tightened by four screws through a rubber filling. The plexiglass wall ensure the fixing of the control thermometer inside

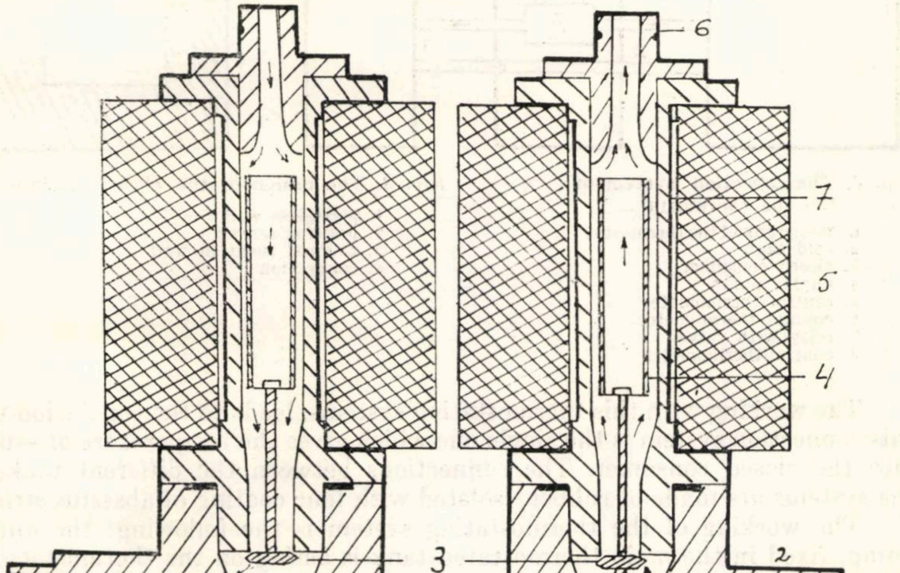

Fig. 2 The design of the relay with valve

1. cylinder of brass
2. connection tube
3. valves
4. cylinder of iron
5. spools
6. connection pieces with double thread
7. cylinder of action

the thermstated bath. In the same time it ensure the posibility of the following of the thermostated liquid.

The description design is represented in fig..3.

It can be observed that the relay is connected between the two connection tubes of the cold tank. Also, it can be seen that the connection tubes between the duplex pump and relay is made to take into consideration the single possible direction of circulation of the thermostating liquid through relay.

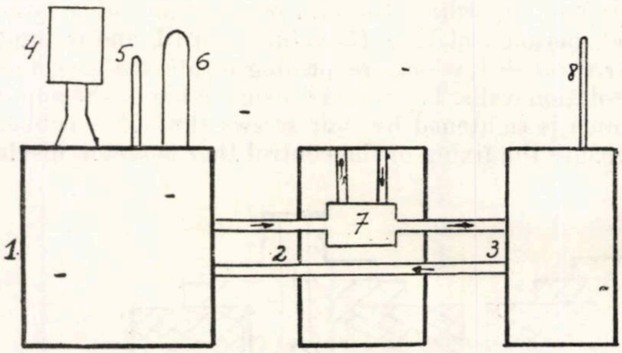

Fig. 2. The conection system of the the thermostat

I. main tank of the thermostat
2. cold tank
3. closed consumer
4. duplex pump
5. control thermometer
6. contact thermometer
7. relay with valve
8. control thermometer

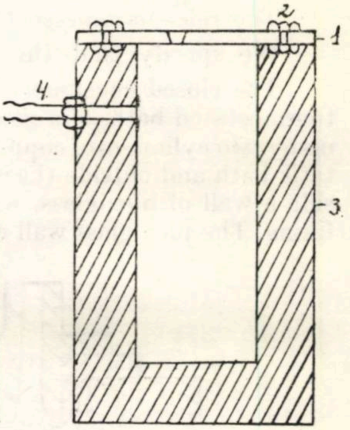

Fig. 3. The design of the closed consumer

1. plexiglass wall = 8
2. pressing screws
3. thermal isolation wall
4. connection

The working with this thermostating systems, leads to the conclusion that this connection system is the single one which gives the temperature of —60°C into the closed consumer. The connections between the different tanks of the systems are made of rubber, isolated with four coating of absestos strings.

The working of the thermostating system is the following: the duplex pump, fixed in the main thermostated tank is acting on the thermostated liquid, which is reaching the relay under pressure and of a flow capacity of 6 L/minutes. If the relay is in action, the thermostated liquid is going through the connectin tube in the second cylinder of brass and reach the closed consumer. The pump is absorbing this liquid which is going again into the main thermostated tank. If the relay is not in action, the thermostated liquid is going through the cylinder of brass, the connections pieces with double thread, the tube of rubber thermal isolated reaching the cold tank. From the cold tank, the thermostated liquid is lead through the second cylinder of brass, the closed consumer where it makes a fast fall of temperature. From the closed consumer the thermostated liquid, under the action of the pump is reaching the main thermostated tank.

The action of the relay is controlled by a contact thermometer through the main relay of the thermostat.

The circulation of the thermostated liquid on the following way: the main tank-relay with valve — cold tank — closed consumer — main tank gives the time of the falling of the temperature. The circulation of the thermostated liquid on the way: main tank — relay with valve main tank — gives the time of the raising of the temperature. The high accurancy constant temperature is achieved when the two times are the same; this can be proved by the lighting of the controlling lamp.

THE DISCUSION OF THE RESULTS OBTAINED

The variation of the main parameters, which are describing the thermostat is represented in the following figure.

În fig. 5a there is a description of the time of obtaining the temperature of —60°C, comparatively with the thermostats NB and U_6. It can be seen that in no more that 20 minutes it is possible to obtain a temperature of —50°C; after this point the experimental curve is changing its slope, so that a temperature of —60°C, can be achieved with this thermostat in I hour. We can mention that this time of obtaining the lowest temperature can be made shorter with some slighty changes of the thermostated circulation.

În Fig. 5b there is a representation of the consumption of solid CO_2 used for the obtaining the desired temperature.

În Fig. 5c is represented the solid CO_2 consumption used for maintaining a constant temperature at the maximum accuracy with the cold tank full of cold agent.

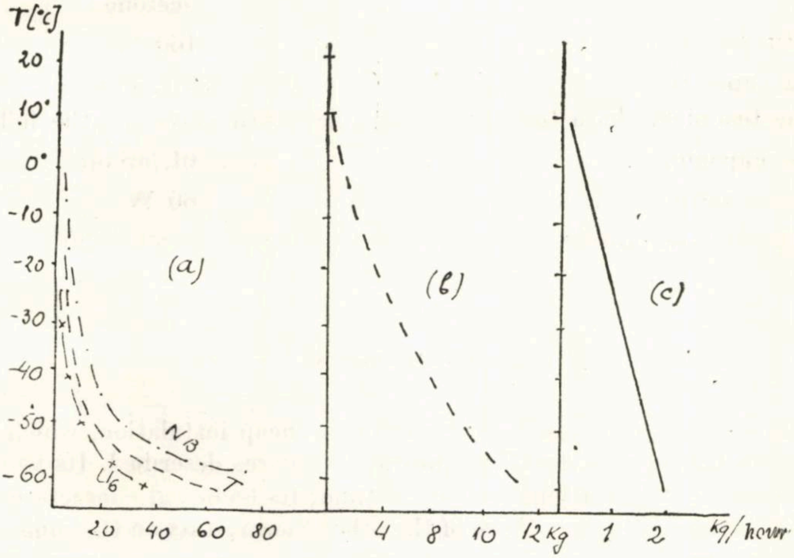

Fig 5 The variation of the main parameters of the thermostat

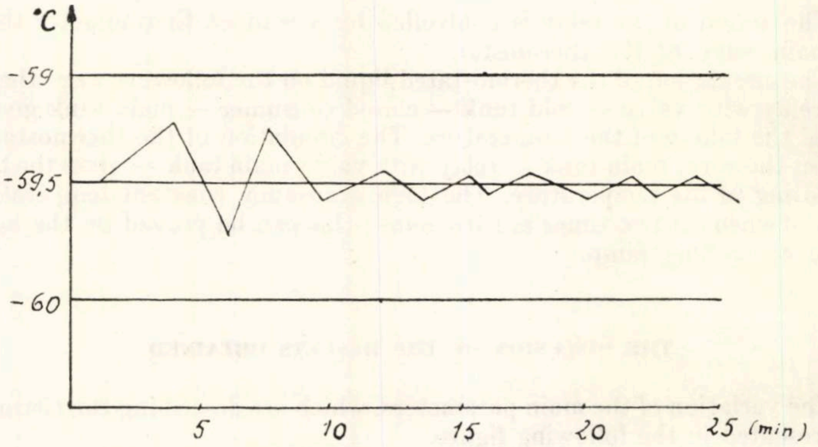

Fig. 6. The experimental curve obtained for the maintaining of a constant temperature at −59,5°C.

În Fig. 6 is designed the experimental curve obtained for the maintaining of a constant temperature of −59,5°.

The technical characteristics of the thermostat are the following:

1. Range of temperature	−60, +20°C
2. Accurancy of maintaining a constant temperature	0,5°C
3. Cold agent	solid CO_2+ acetone
4. Power consumption	160 W
5. Flow capacity	6 L/minute

The technical characteristics of the relay with valve are the following:

1. Flow capacity	6L/minute
2. Power consumption	60 W
3. Range of temperature	−60, +20°C

CONCLUSIONS

This thermostat of low temperature is a cheap instalation, which can be built up by fitting to a usual thermostat the pieces described. Its working is simple, without fail over long periods of time; its technical characteristics are in the same time as high as those of the other thermostats on the same domain of temperature.

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DISPLAY BASIC AND A CLASS OF ITS APLICATIONS USING A HEWLETT-PACKARD 2116 COMPUTER *

MIHAIL LUCIAN PASCU, BRIAN W. JOHNSON

Content

A review of BASIC formal language and of DISPLAY BASIC variant is done. The HEWLETT-PACKARD 2116 computer is introduced. The curves of the distribution of the density of probability of the angular location for the electron of the hydrogen like systems are computed using DISPLAY BASIC programming and the H.P. 2116 computer. Some of the realised programs are presented.

1. INTRODUCTION

Generally speaking, to communicate with a computer in order to introduce data, programs, etc. in it and to obtain computed information from it, formal languages are used.

A formal language is a hybrid language constructed as a combination of the spoken language and algebra, being characterized consequently by some grammatical rules on one hand and by algebraic rules on the other one.
BASIC is such a formal language used especially to communicate in „real-time“ with small computers. For instance the first listed program (Table Section 4) — written essentially in BASIC is a program realised by the authors in order to compute and display the $|Y_{1,1}(\theta, \varphi)|^2$ function using 512 points. The function $Y_{1,1}(\theta, \varphi)$ is the associated Legendre polynomial [3] for $l = 1$, $m = 1$, solution of the angular part of the Schrödinger equation of the hydrogen like systems and has the normalized expression given in table 19 — Section 4. From the point of view of the grammatical rules, this program is realised using words which keep, the meanings they have in the usual language (LET, FOR, NEXT, END). There are also shortened forms of words used in the spoken language like DEF (from DEFINE, DEFINITION) and some other words like CALL which have the same form as the common words but in spite of the

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fact that they give specific instructions to the computer, do not have, necessarily, the same meaning.

More than that, it is possible to write such programs that the computer is capable of reproducing any expression or phrase of the usual language. The program written in Table 20, for instance, (Section 4) is realised for computing the refractive index of a medium N2 from the refraction low, using the expression

$$N2{=}N1*SIN(X)/SIN(Y) \tag{1}$$

where X, Y, and N1 are respectively, the incidence and refraction angles — in radians — and the refractive index of the first medium.

When this program is run, the phrase written in the first line will be printed each time at the input/output unit of the computer. In the second line of this program the input variables N1, X, and Y are separated by commas like in algebra.

At the same time from any of the programs given in Tables 1 to 18 (Section 4) one can see some examples about the algebraic rules used in BASIC. First of all, the following arithmetic operations are defined: exponentiation — symbol "↑“ — , multiplication symbol "*“ — , division — symbol „/“ —, addition-symbol „+“ — and the subtraction — symbol „—“. Their order of precedence is like in algebra, exponentiation, multiplication and division, addition and substraction. This order of precedence is supposed in the lines 120 and 130 of the program given in Table 2. In the case of a more complicated expression, like the one used in Table 16 line 100 for $|Y_{4,2}(\theta, \varphi)|^2$, namely:

$$|Y_{4,-2}(\theta, \varphi)|^2 = \frac{45}{128} \cdot \frac{1}{\pi} \cdot (-\sin\theta)^4 \cdot [(\cos^2\theta - 1)^3 + 6\cos^2\theta\,(\cos^2\theta - 1)^2]^2 \tag{2}$$

where $\theta \equiv X$, in addition to the precedence rule outlined above, the precedence rule of parentheses — which tells that the order of priority of computing expressions into the parentheses decreases from inside to outside — is used.

DISPLAY BASIC uses the same kind of language as BASIC. The only difference between them is the capability DISPLAY BASIC has to permit the computation — point by point — of different curves and to display the points on the screen of the DISPLAY UNIT used as in Fig. 1 as an output unit [2]. This capability is due to DISPLAY subroutines added in such a way that the particularities of the programs written in BASIC are kept fot the display programs. Some of the characteristic elements of BASIC and DISPLAY BASIC programs are presented in the next section.

2. ELEMENTS OF BASIC AND DISPLAY BASIC

To begin with BASIC, one can define [1] the STATEMENT (LINE) which is the essential element of a program (ex. any of the programs given in Tables 1 to 18, 20). A STATEMENT begins with a STATEMENT NUMBER which may be an integer from 1 to 9999 for the H.P. 2116 computer. The statements are arranged in a numerical order from smaller to larges statement num bers no matter what order the statements were entered into the computer. The succesion of statements in the program might be for statement numbers incrementing by 1 but usually the statements are numbered by 10 in order to permit insertion of some additional statements, if necessary (see, for instance Table 3 and Table 20 in section 4).

The next part of a statement is the INSTRUCTION it gives to the computer. The instruction informs the computer about the kind of operation it has to do, and names the type of statement. The instruction acts upon an OPERAND which is a number, or expression or variable, these last terms being introduced in BASIC with the following meanings:

– VARIABLE or SIMPLE VARIABLE is a letter from A to Z or a letter followed by a digit from O to 9; it denotes a numeric value which may be the value of a number or of an algebraic expression or, particularly, of a constant. In Table 4, J is a variable but A,B,C, . . . D1, F1, H1, etc. may be considered also variables.

– The NUMBER is interpreted as a decimal number between 10^{-38} and 10^{38}, zero included (like 3.14, 512, etc. from Table 4). The maximum number of decimal digits for expressing numbers is 6 for the H.P. 2116 computer, so that for high enough or small enough numbers the E notation is used, which expresses the number with more than six significant decimal digits in the form of a decimal number with no more than six significant decimal digites multiplied by 10 to some appropriate power. For instance, 1.0031E+8 is in BASIC equal with 1.0031*10↑8, i.e. 100,310,000; or 1.0031E–8 = 1.0031*10↑(–8)= = 0,000000010030.

– The term OPERATOR is used to represent an arithmetic operation, to assign a value to a variable or to determine the logical relationship between two quantities. (this last field is not included in the purpose of this paper). The ASSIGNEMENT OPERATOR has the symbol „=“ and assigns a value to a variable, having the meaning of „takes the value of“ (for instance the „=“ in the statement 30 of the program given in Table 5). ARITHMETIC OPERATORS have the symbols (like those for arithmetic operations)↑,*,/+ +, – and the same order of precedence as the arithmetic operations.

– The EXPRESSION may be defined now as a combination of variables constants and operators evaluating a numerical value. In Tables 5 and 6, F and F1 name expressions, for instance.

The entities introduced above are usually operands in the expression of a statement, particularly of the statements of the programs presented in this paper (Tables 1 to 18, 20). In these programs the statement numbers are represented by integers from 10 to 200. The instructions are usually written after one blank space after the statement number with the operands at a given distance from instructions. The separations are used in order to make the reading of the programs easier. BASIC is a so called fre-format language in that the distances between the different words and expressions in one line are not essential (the line 70 from Table 3 is equivalent with the line 70 from Table 4). In order to understand better how the programs work, the appropriate instructions of BASIC and DISPLAY BASIC are introduced in the following:

– Instruction LET assigns to a variable a new value, i.e. that of another variable or an expression or a number. In line 30 of the program in Table 9, is the variable to be assigned the value and 35/(64*3.14) this value, which is a number and is assigned using the ASSIGNEMENT operator.

În line 100 of the program in Table 14, the LET instruction assigns the value of the expression (A*(C↑2—1)↑4*(C↑2)/(D4)) to the F variable.

— Instruction DEF FN makes possible the definition of a large variety of functions within the program. In fact, the instruction DEF represents generically instructions of the type DEF FN single letter from A to Z (simple variable), like the instructions DEFA FNA(X) and DEF FNB (X) in lines 40 and 50 of the program in Tables 1 to 18. The functions to be defined can be either any expression introduced in the program or functions predefined in the computer. The predefined functions for the H.P. 2116 computer are common mathematical functions or common trigonometric functions as: ABS (expression) — the absolute value of the expression within the parentheses.

ATN (expression) — the arctangent of the expression within the parentheses (gives the angle in radians)

COS (expression) — the cosine of the expression within the parentheses (the expression must be given in radians).

EXP (expression) — exponentiation of e to the power of the expression's value.

INT (expression) — the largest integer in the expression within the parentheses.

LOG (expression) — natural logarithm of the positively valued expression within the parentheses.

RND (dummy argument) — a random number between 1 and 0.

SIN (expression) — the sine of the expression given in radians.

SQR (expression) — the square root of the positively valued expression within the parentheses.

TAN (expression) — the tangent of the expression in radians.

In lines 40 and 50 of the programs given in Tables 1 to 18 the DEF FNA (X) and DEF FNB(X) instructions define for the program predefined functions. The functions predefined this way may be used as expressions in LET statements, like in lines 80 and 90 of the program in Table 15, or may be used as parts of expressions as in line 100 of the same program.

— Instruction GO TO permits the transfer of control of the computer to the statement specified by the statement number given by the GO TO instruction. In the following segment of program line 50 causes after the computing of the sequence from 22 to 47, the transfer of control to statement 22, i.e. to compute once mose the 22 to 47 sequence with some other values of data:

```
⋮
22 DEF FNP(X) = SIN(X)/(3.14*COS(X)
⋮
47 PRINT "B = "B
50 GO TO 22
⋮
500 END
```

— Instruction IF . . . THEN is used in a program to transfer control to given statement if a specified condition is true. So, in line 25 of the program given in Table 20, the IF . . . THEN instruction used, states that if the value of the computed variable N2 is smaller then 0.9, then control must be transfered to the line 2 in order to input some other values of N1,X and Y simple variables. N2 smaller then 0.9 represents here the condition to be true and therefore a transfer of control to line 2. If this condition is false the computer skips line 25 and executes line 30 of the program.

— Instruction FOR . . . NEXT: the GO TO causes a repeat of a group of statements permanently, unless the programm is stopped by the programmer at a given moment of time. When the IF . . . THEN instruction is used the computer repeats the computation of a segment of the program only if a condition is true and does not repeat the segment if the condition is not true. The FOR . . . NEXT instruction allows the repetition of a group of statements within the program to be controlled, i.e. to be performed a controlled number of times. The FOR part of the instruction must be followed by the simple variable with its initial and final values and is introduced in the program at the beginning of the segment to be repetead. The NEXT part of the instruction followed by the simple variable is introduced within the program at the end of the segment. In such a manner is defined a so called FOR . . . NEXT loop within the program, which is repetead a number of times equal with the number of values the simple variable in the FOR part takes between its initial and final values. Such a loop is presented in all the programs of Tables 1 to 18 between lines 70 and 180. In this loop the values of the expressions are successively computed for J (simple variable) = 1,2, . . . 256, i.e. for integer values of J increasing by one from the initial to the final value. So, within the loop of the Table 16 program the values of F, G, H, 128+500*H, 128+500*G, Cl, Dl, Fl, G1, H1, 128 + 500*Hl and 128+500*Gl are all computed first for J = 1 then J = 2 and so on until J = 256, the control then being transferred after the last computation to the next line (190) in order to execute it. It is also possible to specify the step of the variable J from one computation to the next, using the FOR . . . NEXT loop with the STEP instruction as in the following sequence:

```
⋮
190 FORJ = 1 TO10 STEP.01
⋮
250 NEXT K
⋮
500 END
```

J takes in this case the succesive values 1,1.01, 1.02, . . . 10. If the step is not specified, the computer assumes a step of one as in the listed programs.

— Instruction CALL — is an instruction which transfers control to a specified assembly subroutine added to BASIC for some specialized configurations like display, for instance. The CALL statement permits subroutines to be executed without the constraints of BASIC and gives control of the computer to the executed subroutine so that the assembly language subroutine has the capability of altering any portion of the system, including BASIC The CALL instruction has as operands first of all the the identification of the subroutine accessed which identifies the CALL function and secondly a a parameter which list is a function of the specified subroutine and its purpose. For display, for instance, there are three kinds of statements each of them with well defined functions [2].

The first type of CALL statement is CALL (1, N, X, Y) which is a statement for determining the position of a point on the screen of the DISPLAY UNIT, by computing the components of the position vectors points on the x and y axes of the screen, where (O, O) is the origin of the coordinates located in the extreme left corner of the bottom of the screen. This way it is possible to compute point by point a curve starting from its equation. In the expression of the CALL instruction written above, 1 gives the type of the instruction; N references one of the total number of points M, which are at the disposal of the programmer to plot a curve and is a function of the particular problem and of the desired density of points on the screen; X, Y are the coordinates of the point N in the succession of points which are plotted. For each problem a number of (X, Y) pairs is plotted equal with the number of the computed points (i.e. equal with M). When the program is run and the type 1 CALL statement is introduced within a FOR . . . NEXT loop, N takes values stepwise from its smallest value to the greatest one,indicating eventually the order ((place) in the whole range of each point computed and plotted. It is not necessary that the first value of N be 1, but if is different from 1, the points between 1 and the initial value of N in the particular problem have before the plotting some random positions which are kept and plotted thereafter It is not necessary that (X, Y) values be related with N values, either, but it is convenient sometimes to relate the successive values of at least one of these coordinates with N's values because this assures an automatic change of the value of the respective coordinate stepwise by a step which is function of the step within the FOR . . . NEXT loop. The origin of the coordinate system can be choosen so that the first point be computed in a desired position.

The second type of CALL statement is CALL (2,M) which is a statement for displaying the DISPLAY UNIT. Number 2 gives the type of the CALL statement whereas M tells the display driver how many memory words (points) are to be displayed. This number M may be different from N because it is always possible to introduce within a program before a CALL (2,M) statement two or more CALL (1,N,X,Y) statements so that points are computed for all N's $\leqslant$ M.

The third type of CALL statement for display purposes is CALL(3) which essentially turns off the display, i.e. blanks the screen of the DISPLAY UNIT.

A particular case of the way these CALL statements work is shown in any of the programs given in the tables 1 to 18. Within the program of the Table 16, a program for computing the polar diagram of the function $|Y_{4,-2}|^2$ in lines 130 and 170 there are type, CALL statements. For the CALL statement of line 130, J replaces N of the previous description, 128 + 500∗H is the expression of the x coordinate and 128 + 500∗G is the expression of the y coordinate. The statement belongs to a FOR . . . NEXT loop beginning with

J = 1 and ending with J = 256. The origin of the coordinate system is the same with J = 256. The origin of the coordinate systems is the same with that previously described but the positions of the points are computed around the center of the screen coordinates (128, 128) because the resolution of points in the x and y directions is 256 on the screen). The plot is a polar diagram so that the F expression introduced in the line 10) after the equation (2) is considered the absolute value of the position vector of each point, and changes together with, the angle, when θ varies from O to 3.14 ($\pi \equiv 180°$) to 6.28 ($2\pi \equiv \equiv 360°$) by a step of 3.14/256. In fact semicircle is divided into 256 parts, the constant B, introduced in line 60 giving the value of each part, namely 3.14/256. The variables C and D introduced in the FOR . . . NEXT loop in lines 80 and 90 represent respectively cos and sin of the variable J∗B. Each time J changes its value at the begining of the loop, C and D change their values corresponding to the new J∗B value of their argument and consequently F changes its value. In this way there are 256 values of the angle between O and 3.14 radians, and 256 values of F in the same range. H and G represent the projections respectively of F on the X and Y axes and change their values after lines 110 and 120 first because of the F changing and secondly because of the C,D, changing. They have each 256 values for $\theta \in [0, 3.14]$. The factor 500 which multiplies the values for H and G is a scale factor permitting the establishment of the convenient dimensions of the curve plotted around the (128, 128) point. The CALL statement introduced in line 170 continues the same kind of computations for $\theta \in [3.14, 6.28]$ using another 256 points for this part of the circle, the points numbered 257 to 512. After the end of the loop there are 512 computed points which are displayed when statement, 190 is executed; on the screen appears the curve given in Fig. 3 and the program is terminated. In all cases after execution of the CALL statement is finished, control is transferred to the statement immediately following the CALL statement.

— Instruction PRINT causes the value of the expression of the value of the variable written in the place of its operand to be printed at the OUTPUT UNIT (see section 3). In the program of Table 20 for instance when line 22 is executed the most recent computed value of N2 is printed at the OUTPUT UNIT and may be known immediately after its computation.

— Instruction INPUT is used in order to assign a value directly from the INPUT UNIT (see section 3) to a variable within the program. For instance, each time, control is transferred to line 2 of the program given in Table 20 the operator must introduce the values desired for N1, X,Y in order to obtain finally the computed value of N2. The computer stops asking for new input values when $N2 < 0.9$.

— Instruction REM makes possible the introduction as its operand of a comment or a remark or of any other expression or succession of characters within the program, and causes the listing of the operand each time the program is listed or recorded. In the program of Table 20 line 1 is a REM line and shows the type of the program which follows. REM statements are not executed when the program is running, they are only comments to the program.

The instructions described above are present in a general type statement and act on a given operand. There is also a category of statements which do not contain operands but only statement numbers and commands. In this category is the END statement whose purpose is to inform the computer that the program is complete when it is introduced in the machine and to terminate the execution of the program when this one is run. The END statement must be consequently the highest numbered statement in the program, as can be seen in all the programs presented in this paper.

As a conclusion of this section, the program might be defined as a succession of statements, the highest numbered statement being necessarilly an END statement.

3. INTRODUCTION OF THE HEWLETT-PACKARD 2116 COMPUTER

The computations and results reported in this paper were done using a Hewlett-Packard 2116 computer, whose block diagram is shown in Fig. 1.

The INPUT UNIT is the part of the computer which permits the programmer to introduce programs, data, and commands to the computer. The OUTPUT UNIT makes available computed information and replies to commands. Usually, these two units are condensed in an INPUT/OUTPUT UNIT which performs both functions of inputting) outputting information from the computer. Input devices are, for instance, for the H.P. 2116 computer, the teleprinter, high speed photoelectric tape reader and keyboard display terminal (the information may be introduced usually on punched tape or directly using the teleprinter or keyboard display terminal); output devices are the teprinter, line printer, tape punch, magnetic tape units, X—Y recorder, etc. Other input devices may be measuring instruments which provide a digital data output like digital voltmeteres followed by signal converters for measuring DC and AC voltages, currents, etc., electronic counters for frequency or period measurements, scalar timers, etc.

A particular type of OUTPUT UNIT is the DISPLAY shown in Fig. 1 which us used in order to display point by point computed plots or curves. The capabilities of the H.P. 2116 computer and of the associated DISPLAY UNIT used for the computations done in this paper (the display channel is monodirectional) are related first, with the total number of points available for display, and secondly with the total number of points on the x and y axes of the screen of the DISPLAY UNIT. The total number of points available for display is in this particular case 1000 and the total number of points available on the x or y axes are 256. That means, the computer may use a maximum number of points of 1000 in order to plot a curve in the x, y plane of the screen of the DISPLAY UNIT. It uses also a total number of 256 points in order to display a line parallel with the x or y axes on the same screen. The computed data available to the DISPLAY UNIT are provided either directly from the ARITHMETIC UNIT or from the MEMORY UNIT.

The ARITHMETIC UNIT performs the computations of the programs and manipulates the data making them available for memory or for output directly. The basic operations performed by the computer in fact by its ARITHMETIC UNIT are outlined in section 2; in that same section are introduced also the predefined functions for the H.P.2116 computer. The main

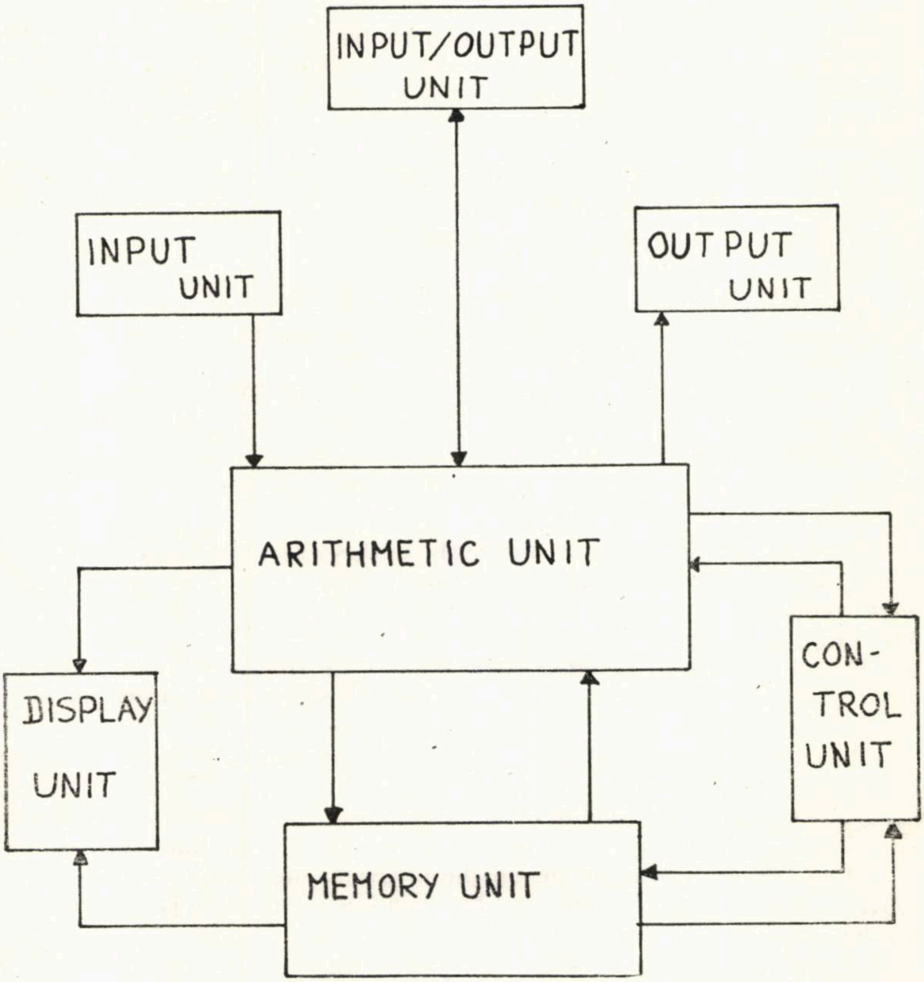
INPUT/OUTPUT UNIT
INPUT UNIT
OUTPUT UNIT
ARITHMETIC UNIT
DISPLAY UNIT
CON-TROL UNIT
MEMORY UNIT

Fig. 1

purpose of the ARITHMETIC UNIT is to compute the values of the different expressions introduced within the programs using the instructions given in the programs in order to make possible the computations with the operations and functions known by the computer.

The MEMORY UNIT is a ferrite core memory which permite the simultaneous storage of 8192 word, each of them containing 16 bites of information. The quantity of stored information available to the computer may be much greater if the MEMORY UNIT is completed with some external memory systems (magnetic tapes, etc). The MEMORY UNIT defined, makes available to any other unit of the computer, particularly to the OUTPUT UNIT, any computed information or any other necessary information it stores. The transfer of data between the memory and the other units within the computer and between any other units within the computer is controlled by the CONTROL UNIT. The main connections between the above described parts of the H.P. 2116 computer are shown in Fig. 1 and the results obtained with it in the calculations done in this paper are reported in the next section.

4. RESULTS

The Schrödinger equation for hydrogen-like systems is solved supposing that the wave function $\Psi_{n,l,m}(r,\theta,\varphi)$—where r,θ,φ are the spherical coordinates: $r \in [0, \infty]$, $\theta \in [0,\pi]$, $\varphi \in [0,2\pi]$ describing the behaviour of the system may be written as a product of two functions: $R_{n,1}$, (r) the hypergeometric function — the radial part — and $y_{1,m}(\theta,\varphi)$, the associated Legendre polynomials, — the angular part —:

$$_{n,l,m}(r, \theta, \varphi) = R_{n,1}(r)\, Y_{1,m}(\theta, \varphi) \tag{3}$$

The $| Y_{1,m}(\theta, \varphi) |^2$ quantities describe in this way the density of probability of angular location of the electron around the nucleus. Their polar diagrams were computed and obtained on the DISPLAY UNIT using DISPLAY BASIC programming and the H.P. 2116 computer. The expressions of the $Y_{1,m}(\theta, \varphi)$ functions are given in Table 19, and the programs for computing the polar diagrams in Tables 1 to 18; the way these programs work is pointed out in section 2.

Comparing the programs one observes that their algorithm is the same, but with some differences, due to the manner the programs were computed, being present as the numbers of some lines are concerned. Standardizing this notation, i.e. supposing that between the line 10 and the line 70 the lines 30, 50, 50 and 60 are processed for all the programs and supposing also that the FOR . . . NEXT loops is computed between line 70 and line 170, one can draw the logical schematic for such a type of program.

În Fig. 2 the FOR . . . NEXT loop is represented by the processing rectangle for lines 70 to 170, the decision diamond and the processing rectangle for adding one unit to J. The display operation which follows represents, in fact, line 190 of the programs. The schematic is limited by the START and END statement ovals indicating the beginning and the ending points of the programs. The results computed and displayed using this kind of programs are given in Fig. 3 for $Y_{1,m}(\theta, \varphi)$ functions with $1 = 1,2,3$ and m having the corresponding values (0,1 . . . 1) and $1 = 4$ and $m = -2, -3, -4$ only. There were obtained the known polar diagrams which describe the angular geometric properties of the electronic cloud of the hydrogen-like system. The density of probability of the angular distribution of the electron within the electronic cloud is maximum in the planes predicted by the Bohr-Sommerfeld theory.

The pictures are images of the curves obtained (using 512 points as outlined in section 2) on the screen of the DISPLAY UNIT.

Table 1

```
 10 CALL(3)
 30 LET A = 3/(8*3.14)
 40 DEF FNA(X) = COS(SX)
 45 DEF FNB(X) = SIN(X)
 50 LET B = 3.14/256
 60 FOR J = 1 TO 256
 70 LET C = FNY (J*B)
 75 LET D = FNB(J*B)
 80 LET F = A*(D↑2)
 90 LET G = F*C
100LET H = F*D
110 CALL (1, J, 128 + 500*H, 128 + 500*G)
120 LET C1 = FNA (3.14 + J*B)
130 LET D1 = FNB(3.14 + J*B)
140 LET F1 = A*(D1↑2)
150 LET G1 = F1*C1
160 LET H1 = F1*D1
170 CALL (1,J + 256, 128 + 500*H1,
  128 + 500*Gl)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 2

```
 10 CALL (3)
 30 LET A = 3/(4*3.14)
 40 DEF  FNA(X) = COS(X)
 45 DEF FNB(X) = SIN(X)
 50 LET B. = 314/256
 60 FOR J = 1 TO 256
 70 LET C = FNA (J*B)
 75 LET D = FNB (J*B)
 80 LET F = A*(C↑2)
 90 LET G = F*C
100 LET H = F*D
110 CALL (1, J, 128 + 400*H, 128 + 400*G)
120 LET C1 = FNA (3.14 + J*B)
130 LET D1 = FNB (3.14 + J*B)
140 LET F1 = A*(C1↑2)
150 LET G1 = F1*C1
160 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 400*H1,
  128 + 400*G1)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 3

```
 10 CALL (3)
 30 LET A = 3/(8*6.28)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FORJ = 1 TO 256
100 LET C = FNA (J*B)
110 LET D = FNB(J*B)
120 LET F = (A*(C↑2−1)↑2)/D↑2)
123 LET G = F*C
127 LET H = F*D
130 CALL (1,J, 128 + 500*H, 128 + 500*G)
140 LET C1 = FNA (3.14+J*5)
150 LET D1 = FNB(3.14+J*B)
160 LET F1 = (A*(C1↑12−1)↑2/(D1↑2)
163 LET G1 = F1*C1
167 LET H1 = FJ 1*D1
170 CALL (J, + 256, H1, 128 + 500*H1
  128+500*G1)
180 NEXT J
190 CALL (2, 512)
200 END
```

Table 4

```
 10 CALL (3)
 30 LET A = 15/(512*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET  B = 3.14/256
 70 FOR JîL TO 256
 80 LET C = FNA (J*B)
 90 LET D = FNB(J*B)
100 LET F = A*D↑4
110 LET G = F*C
120 LET H = F*D
130 CALL(Y,J,128+10000*H,128+200000*G)
140 LET C1 = FNA (3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*D1↑4
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1, J + 256, 128 + 10000*H1,
  128 + 20000*G1)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 5

```
 10 CALL (3)
 30 LET A = 15/(128*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*C 2*D2
110 LET G = F*C
120 LET H = F*D
130 CALL (1, J, 128+5000*H, 128+100000*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*C1 2*D12
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL (1,J + 256, 128 + 50000*H1,
  128 + 10000*G1)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 6

```
 10 CALL (3)
 30 LET A = 5/(16*3.14)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*((3*(C↑2)-1)↑2)
110 LET G = F*C
120 LET H = F*D
130 CALL (1,J, 128 + 150*H, 128 + 250*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 =A*(3*C1↑2)- 1)↑2
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256,128 + 150*H1,
  128 + 250*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 7

```
 10 CALL (3)
 30 LETA = 15/8*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = (A*(C↑2-1)↑2*C↑2)/(D↑2)
110 LET G = F*C
120 LET H = F*D
130 CALL (1, J, 128 + 500*H, 128 + 1000*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = (A*(C1↑2-1)↑2*C1*2)/(D1↑2)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 500*H1,
  128 + 1000*G1)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 8

```
 10 CALL(3)
 30 LET A = 15/(32*3.14)
 40 DEF  FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = (A*(C↑2-1)↑4)(D↑4)
110 LET G = F*C)
120 LET H = F*D
130 CALL(1,J, 128 + 750*H, 128 + 20000*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = (A*C1↑2-1)↑4)/(D1↑4)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256,128 + 750*H1,
  12 8+ 2000*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 9

```
 10 CALL(3)
 30 LET A = 35/(64*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*(D↑6)
110 LET G = F*C
120 LET H = F*D)
130 CALL(1,J, 128 + 650*H, 128 + 650*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB)3.14 + J*B)
160 LET F1 = A*(D1↑6)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 650*H1,
  128 + 650*G1
180 NEXT J
190 CALL(2,512)
200 END
```

Table 10

```
 10 CALL(3)
 30 LET A = 105/(32*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J×B)
100 LET F = A*(D↑4)*(C↑2)
110 LET G = F*C
120 LET H = F*D
130 CALL(1,J,128 + 750*H, 128 + 750*G)
140 LET C1 = FNA(3,14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*(D1↑4)*(C1↑2)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 750*H1,
  128 + 750*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 11

```
 10 CALL(3)
 30 LET A = 21/(64*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(F*B)
 90 LET D = FNB(J*B)
100 LET F = A*(D↑2)*((5*C↑2)-1)↑2)
110 LET G = F*C
120 LET H = F*D
130 CALL(1,J, 128 + 500+H, 128 + 500*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*(D1↑2)*(5*C1↑2)-1)↑2)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 500*H1,
  128 + 500*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 12

```
 10 CALL(3)
 30 LET A = 7/(16*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*(5*C↑3)-3*C)↑2)
110 LET G = F*C
120 LET H = F*D
130 CALL (1,J, 128 + 100*H, 128 + 150*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*(5*(C1↑3)) - 3*C1)↑2
163 LET G1 = F1*C1
167 LET H1 = D1*F1
170 CALL(1, J + 256, 128 + 100*H1,
  128 + 150*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 13

```
 10 CALL(3)
 30 LET A = 21/(64*3.14)
 40 DEF  FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/126
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = (A*(C↑2−1)↑2 + 4*(C↑2 −
  − 1)*(C↑2)↑2)/(D↑2)
110 LET G = F*C
120 LET H = G*D
130 CALL(1,J, 128 + 500*H,128 + 500*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = (A*((C1↑2 − 1)↑2+ 4* (C1↑2 −
  − 1)*(C1↑2)↑2/(D1↑2)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256,128 + 500*H1,
  128 + 500*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 14

```
 10 CALL(3)
 30 LET A = 105/(32*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = DNB(J*B)
100 LET F = A*(C↑2 − 1)↑4*(C↑2)/D↑4)
110 LET G = F*C
120 LET H = F*D
130 CALL(1,J,128 + 750*H,128 + 1250*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1=(A*(C1↑2−1)↑4*(C1↑2))/(D1↑4)
163 LET G1 = F1*C1
167 H1 = F1*D1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 750*H1,
  128 + 1250*G1)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 15

```
 10 CALL(3)
 30 LET A = 35/(64*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = (A*(C↑2 − 1)↑6)/(D↑6)
110 LET G = F*C
120 LET H = F*D
130 CALL(1,J, 128 + 700*H, 128 + 1500*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = (A*(C1↑2−1)↑6)/(D↑6)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 700*H1,
  128 + 1500*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 16

```
 10 CALL(3)
 30 LET A = 45/(128*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 B = 3.14/126
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*((1/((−D)↑4)))*(((C↑2 −
  − 1)↑3 + (6*(C↑2)*((C − 1)↑2)))↑2)
110 LET G = F*C
120 LET H = F*D
130 CALL(1, J, 128 + 500*H,128 + 500*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LETF1 = A*(1/((−D1)↑4)))*(((C1↑2 −
  — 1)↑3 + (6*((C1↑2))*((C1↑2 − 1)↑2)))↑2)
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 500*H1,128 +
  + 500*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 17

```
 10 CALL(3)
 30 LET A = 315/(64*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1 TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*((1/((-D)↑6))*((C↑2 -
 - 1)↑6*(C↑2))
110 LET G = F*C
120 LET H = F*D
130 CALL(1, J, 128 + 500*H, 128 + 500*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*((1/((-D1)↑6))*((C1↑2 -
 -1)↑6)*(C1↑2))
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 500*H1,
  128 + 500*G1)
180 NEXT J
190 CALL(2,512)
200 END
```

Table 18

```
 10 CALL(3)
 30 LET A = 315/(512*3.14)
 40 DEF FNA(X) = COS(X)
 50 DEF FNB(X) = SIN(X)
 60 LET B = 3.14/256
 70 FOR J = 1TO 256
 80 LET C = FNA(J*B)
 90 LET D = FNB(J*B)
100 LET F = A*((1/((-D)↑8))*((C↑2 -1)↑8))
110 LET G = F*C
120 LET H = F*D
130 CALL(1,J, 128 + 500*H, 128 + 500*G)
140 LET C1 = FNA(3.14 + J*B)
150 LET D1 = FNB(3.14 + J*B)
160 LET F1 = A*(((1/((-D1)↑8))*((C1↑2 -
  - 1↑8))
163 LET G1 = F1*C1
167 LET H1 = F1*D1
170 CALL(1,J + 256, 128 + 500*H1,
  128 + 500*G1)
180 NEXT J
190 CALL (2,512)
200 END
```

Table 19

THE ASSOCIATE LEGENDRE POLINOMIALS FOR L=1 TO 4

$$Y_{1,1}(\theta, \varphi) = \frac{1}{2} \cdot \left(\frac{3}{2\pi}\right)^{1/2} e^{i\varphi} (-\sin\theta)$$

$$Y_{1,1}(\theta, \varphi) = \frac{1}{2} \cdot \left(\frac{3}{\pi}\right)^{1/2} \cos\theta$$

$$Y_{1,-1}(\theta, \varphi) = -\frac{1}{2} \cdot \left(\frac{3}{2\pi}\right)^{1/2} e^{-i\varphi} (\cos^2\theta - 1) \cdot \frac{1}{\sin\theta}$$

$$Y_{2,2}(\theta, \varphi) = \frac{3}{16} \cdot \left(\frac{5}{6\pi}\right)^{1/2} e^{2i\varphi} \sin^2\theta$$

$$Y_{2,1}(\theta, \varphi) = -\frac{3}{8} \cdot \left(\frac{5}{6\pi}\right)^{1/2} e^{i\varphi} \sin\theta \cdot \cos\theta$$

$$Y_{2,0}(\theta, \varphi) = \frac{1}{4} \cdot \left(\frac{5}{\pi}\right)^{1/2} \cdot (3\cos^2\theta - 1)$$

$$Y_{2,-1}(\theta, \varphi) = -\frac{1}{2} \cdot \left(\frac{15}{2\pi}\right)^{1/2} e^{-i\varphi} (\cos^2\theta - 1) \frac{\cos\theta}{\sin\theta}$$

$$Y_{2,-2}(\theta, \varphi) = \frac{1}{8} \cdot \left(\frac{30}{\pi}\right)^{1/2} e^{-2i\varphi} (\cos^2\theta - 1) \cdot \frac{1}{\sin^2\theta}$$

$$Y_{3,3}(\theta, \varphi) = -\frac{1}{8} \cdot \left(\frac{35}{\pi}\right)^{1/2} e^{3i\varphi} \sin^3\theta$$

$$Y_{3,2}(\theta, \varphi) = \frac{15}{4} \cdot \left(\frac{7}{30 \cdot \pi}\right)^{1/2} e^{2i\varphi} \sin^2\theta \cdot \cos^2\theta$$

Table 19 – continued

$$Y_{3,1}(\theta, \varphi) = -\frac{1}{16} \cdot \left(\frac{84}{\pi}\right)^{1/2} e^{i\varphi} \sin\theta\,(5\cos^2\theta - 1)$$

$$Y_{3,0}(\theta, \varphi) = \frac{1}{16} \cdot \left(\frac{7}{\pi}\right)^{1/2} \cdot [12 \cdot (\cos^2\theta - 1)\cos\theta + 8\cos^3\theta]$$

$$Y_{3,-1}(\theta, \varphi) = -\frac{1}{8} \cdot \left(\frac{21}{\pi}\right)^{1/2} \cdot e^{-i\varphi} \cdot [(\cos^2\theta - 1)^2 + 4 \cdot (\cos^2\theta - 1)\cos^2\theta] \cdot \frac{1}{\sin\theta}$$

$$Y_{3,-2}(\theta, \varphi) = \frac{1}{8} \cdot \left(\frac{210}{\pi}\right)^{1/2} e^{-2i\varphi} (\cos^2\theta - 1)^2 \cdot \frac{\cos\theta}{\sin^2\theta}$$

$$Y_{3,-3}(\theta, \varphi) = -\frac{1}{8} \cdot \left(\frac{35}{\pi}\right)^{1/2} e^{3i\varphi} (\cos^2\theta - 1)^3 \cdot \frac{1}{\sin^3\theta}$$

$$Y_{4,4}(\theta, \varphi) = \frac{315}{48} \cdot \left(\frac{1}{70 \cdot \pi}\right)^{1/2} e^{4i\varphi} (-\sin\theta)^4$$

$$Y_{4,3}(\theta, \varphi) = \frac{105}{8} \cdot \left(\frac{1}{35 \cdot \pi}\right)^{1/2} e^{3i\varphi} (-\sin\theta)^3 \cos\theta$$

$$Y_{4,2}(\theta, \varphi) = \frac{15}{8} \cdot \left(\frac{1}{10 \cdot \pi}\right)^{1/2} e^{2i\varphi} (7\cos^2\theta - 1)\sin^2\theta$$

$$Y_{4,1}(\theta, \varphi) = \frac{1}{8} \cdot \left(\frac{15}{\pi}\right)^{1/2} e^{i\varphi} [3\cos\theta\,(\cos^2\theta - 1) + 4\cos^3\theta]\,(-\sin\theta)$$

$$Y_{4,0}(\theta, \varphi) = \frac{3}{16} \cdot \left(\frac{1}{\pi}\right)^{1/2} \cdot [3(\cos^2\theta - 1)^2 + 24 \cdot (\cos^2\theta - 1)\cos^2\theta + 8\cos\theta]$$

$$Y_{4,-1}(\theta, \varphi) = \frac{3}{8} \cdot \left(\frac{5}{\pi}\right)^{1/2} \cdot e^{-i\varphi} [3\cos\theta\,(\cos^2\theta - 1)^2 + 4\cos^3\theta\,(\cos^2\theta - 1)] \cdot \frac{1}{(-\sin\theta)}$$

$$Y_{4-2}(\theta, \varphi) = \frac{1}{128} \cdot \left(\frac{90}{\pi}\right)^{1/2} e^{-2i\varphi} \cdot [8\,(\cos^2\theta - 1)^3 + 48\cos^2\theta\,(\cos^2\theta - 1)^2] \cdot \frac{1}{\sin\theta}$$

$$Y_{4,-3}(\theta, \varphi) = \frac{1}{8} \cdot \left(\frac{315}{\pi}\right)^{1/2} e^{-3i\varphi} \cos\theta\,(\cos^2\theta - 1)^4 \cdot \frac{1}{(-\sin\theta)^3}$$

$$Y_{4,-4}(\theta, \varphi) = \frac{3}{32} \left(\frac{70}{\pi}\right) e^{-4i\varphi} (\cos^2\theta - 1)^4 . \frac{1}{\sin^4\theta}$$

$\theta \in [0, \pi]$

$\varphi \in [0, 2\pi)]$

Table 20

```
 1 REM this is a program for
   computing the the refractive index
   of a medium
 2 INPUT N1, X, Y
 5 DEF FNA(Y) = SIN(Y)
15 LET B = FNA(Y)
20 LET N2=N1*A/B
22 PRINT N2
25 IF N2 < 0.9 THEN 2
30 END
```

5. CONCLUSIONS

The reported results show the capabilities of DISPLAY BASIC programming in the description of the geometric properties of the electronic cloud of atoms and molecules once the wave functions for these systems are known It is possible also to compute and plot using such programs curves expressed as a function of the wavefunctions of a system, like ionization cross-sections, etc. Into the expression of the ionization cross-section of an atomic system when the system absorbs a photon[4]

$$\sigma(\nu) = \frac{8\pi^3\nu}{3c} \cdot \frac{1}{\omega_i} \sum_i \sum_f \left| \int \psi_f^* \sum_\mu e_\mu \gamma_\mu \psi_i \, d\tau \right|^2 \qquad (4)$$

— where ψ_i are functions of ω_i — fold degenerate initial state of the system, ψ_f the continuum eigenfunctions of the energy, ν the light frequency, e_μ the charge and γ_μ the position vector, with respect to the center of mass of the μ — th particle of the system — enter the ψ_i and ψ_f functions, for instance. Similarly it might be possible to compute and plot such a curve for the spectral series of alkaly atoms, or for some other appropriate atomic systems.

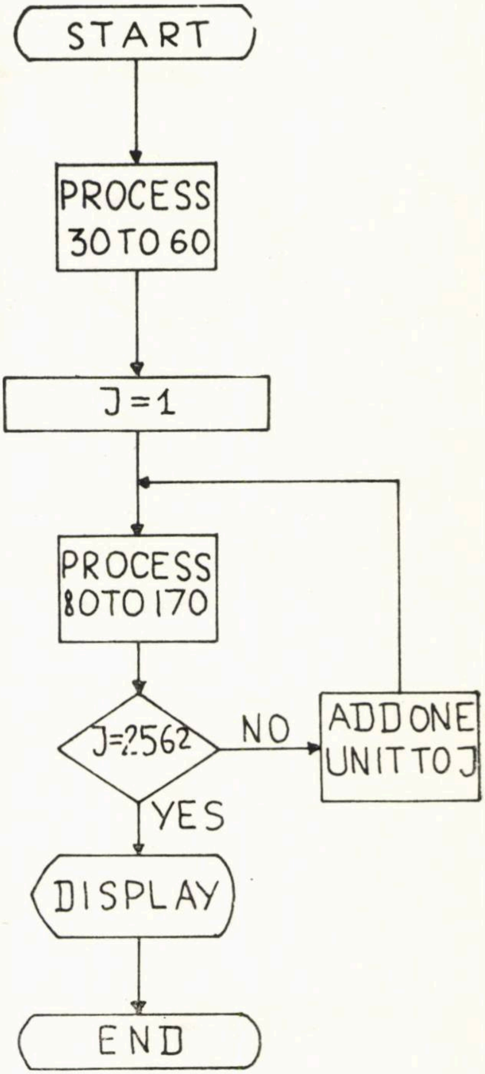

Fig. 2

6. ACKNOWLEDGEMENTS

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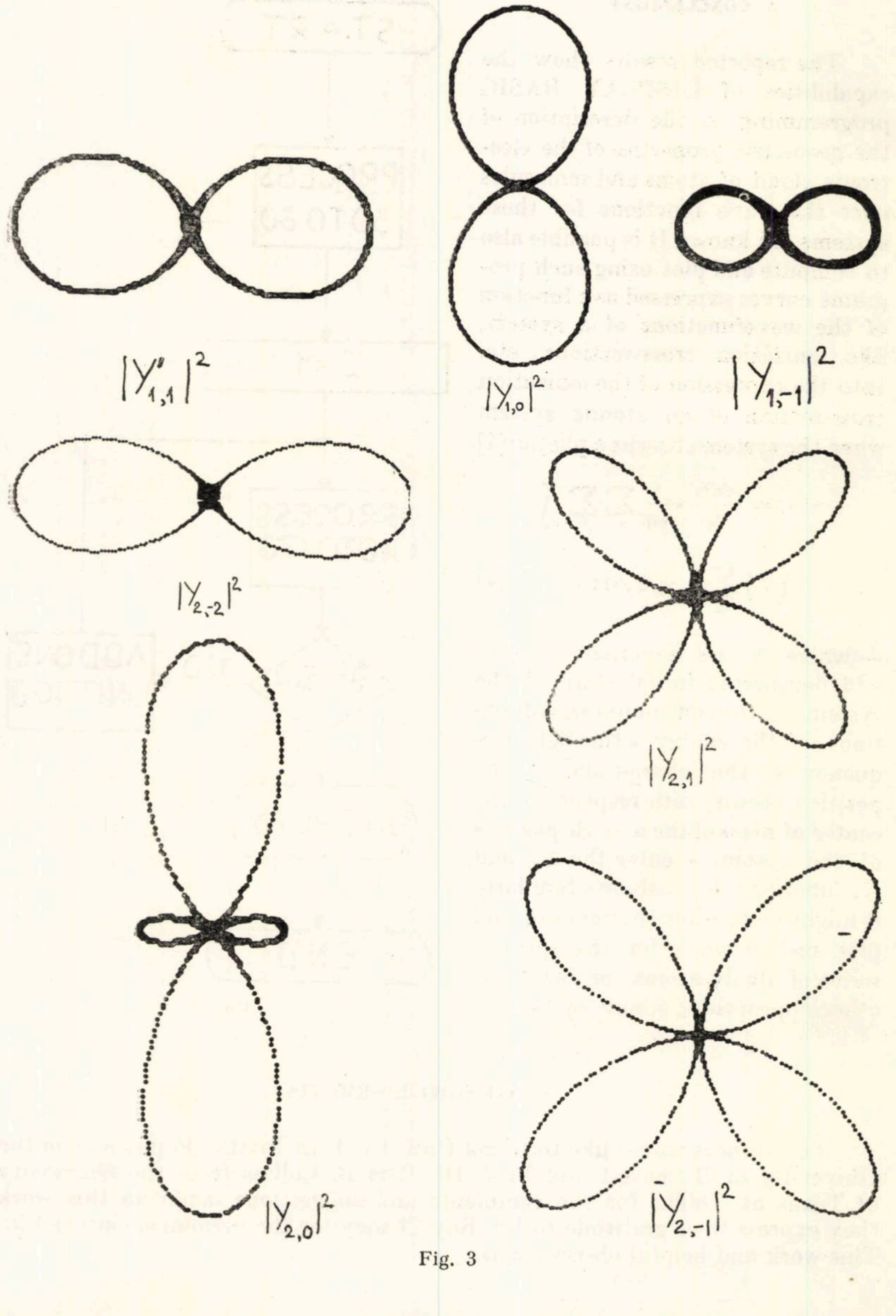

Fig. 3

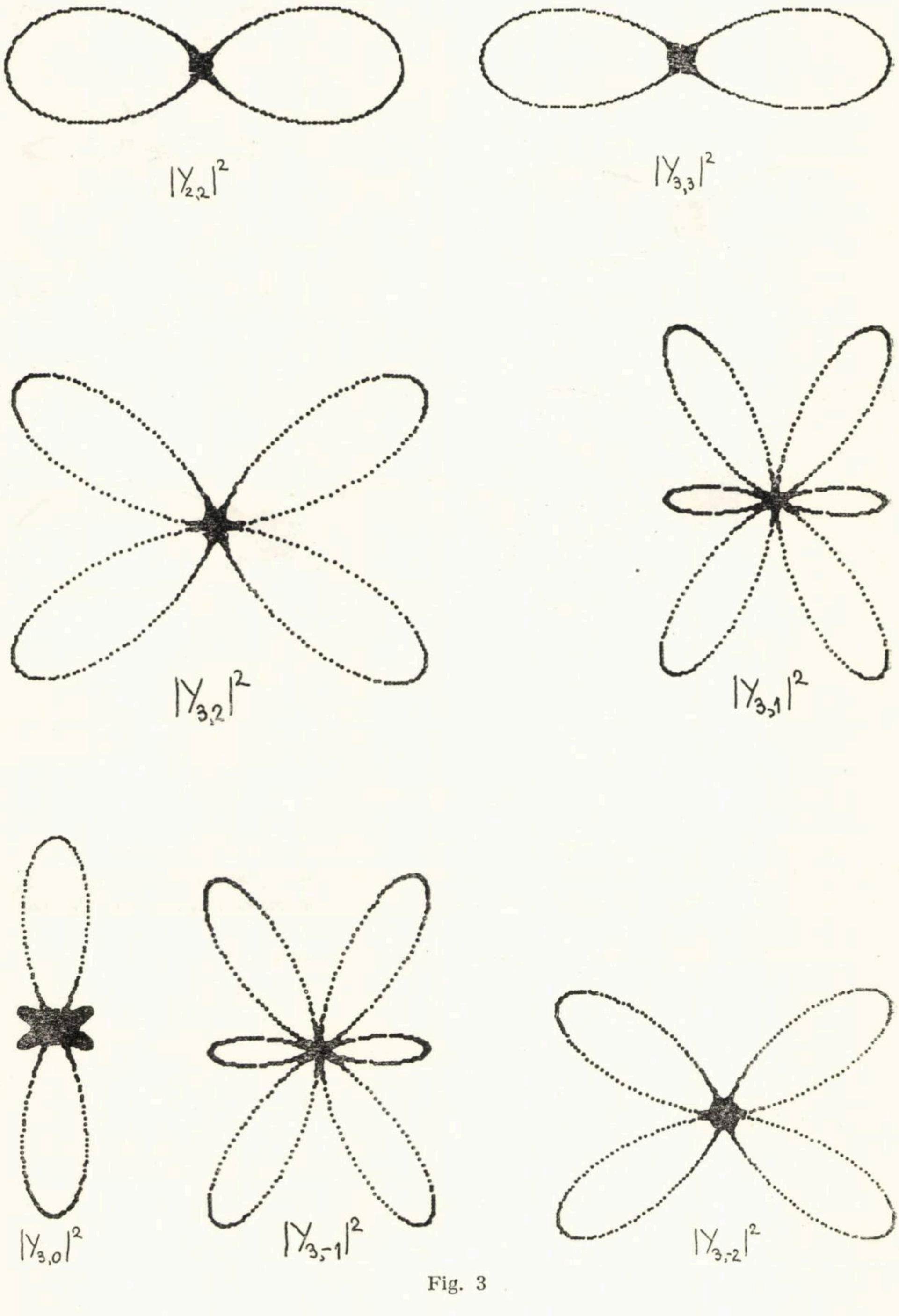

Fig. 3

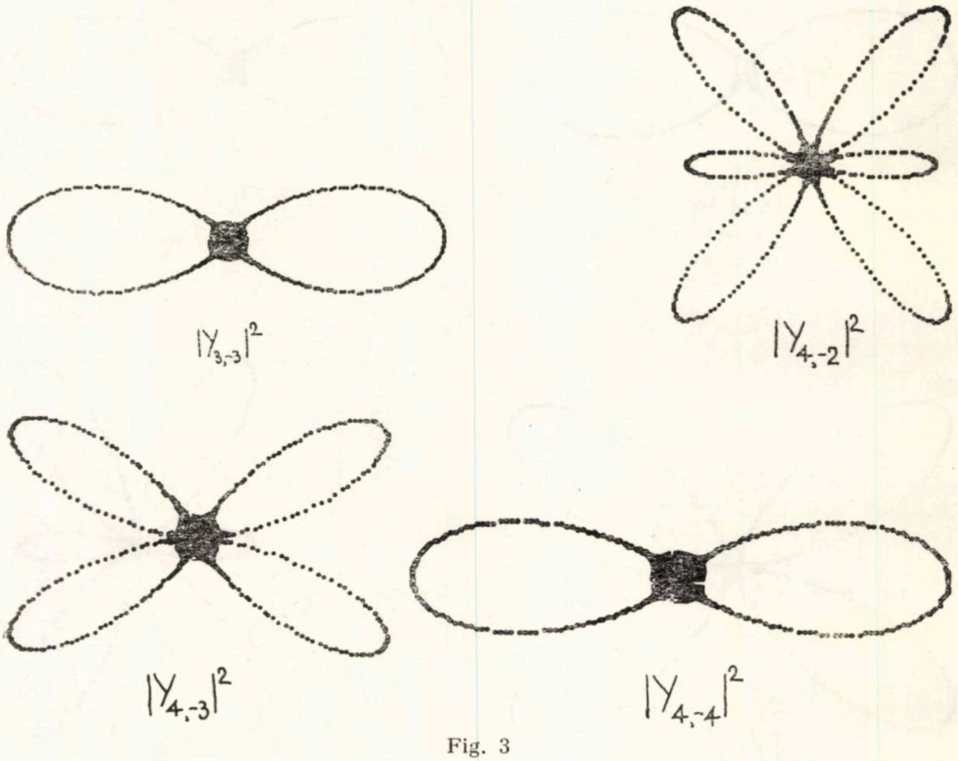

Fig. 3

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COMPUTER AIDED SOLVING OF A TWO TRANSCENDENTAL EQUATIONS SYSTEM REFERRING TO GAS DISCHARGE PHYSICS

AURELIAN IACOV, and ANDREI DRĂGAN,

The elementary theory of the gas discharge ignition phenomenon takes into account an optimum of the ignition field intensity depending on its frequency. The finding out of the optimum calls for the solving of a two transcendental equations system. Since this cannot be done analitically we resort to a computer-aided numerical solving method.

Therefore the compatibility functional must be firstly solved, the computation program being founded on the idea adjusting to an optimal searching step.

A phenomenon of quasi-resonance of the electron flight time justifiying the assumption of the found optimum has been evidenced.

The authors present an original subroutine of graphical curve tracing.

1. THE PHYSICAL RESEARCH WHICH GENERATES THE PROBLEM.

A gas or gas mixture, subject to a continuous bombardment of accelerated electrons is the seat of complex, multiple and various processes of excitation, ionization and recombination which go on uninterruptedly.

The excitation of electrons and ions from the plasma can be done in many ways. Since lately it is frequently resorted to excitation by high-frequency currents, we subsequently present some basic elements on the formation of plasma through discharges in a high-frequency field.

So far no complete theory on discharges in high-frequency fields has been set up, because the mathematical model of the discharge process is difficult to realize.

A simplified theory [2], takes into account the instance of low pressures and relatively high frequencies, when it can be assumed that the electron is moving in a field out of any collision possibility. In this case, his motion equation is given by the dynamic equation of the free material point.

$$m_e \frac{d^2x}{dt^2} = e\ E_m \cdot \sin \omega t \tag{1}$$

where:

m_e — electron mass

x — the position of the electron in respect to a convenient coordinate system Ox

e — electric charge of the electron

$E_m \cdot \sin \omega t$ — the alternating electric field, opulsation ω applied in the Ox direction

The motion law of the electron is arrived at by intergating the equation (1), which results in:

$$V = \frac{dx}{dt} = -\frac{eE_m}{m_e \cdot \omega} \cos \omega t + C_1 \tag{2}$$

$$x = -\frac{e\,E_m}{m \cdot \omega^2} \sin \omega t + C_1 \cdot t + C_2 \tag{3}$$

To determine the integration constants we consider dt $t = 0$, $x = 0$; so that from (3) we have $C_2 = 0$; and the electron comes into the field at a time τ with zero innitial speed, so that from (2) we obtain:

$$C_1 = \frac{e\,E_m}{m_e \cdot \omega} \cos \omega\, \tau \tag{4}$$

Thus, (3) becomes:

$$x = \frac{e\,E_m}{m_e \cdot \omega} \left(t \cos \omega\, \tau \frac{1}{\omega} \sin \omega t \right) \tag{5}$$

It will be seen that the electron motion has two components, one of translation

$$x_{tr} = \left(\frac{e\,E_m}{m_e \cdot \omega} \cdot \cos \omega \tau \right) t \tag{6}$$

and an oscillatory one

$$x_{osc} = -\frac{e\,E_m}{m_e \cdot \omega} \cdot \sin \omega t \tag{7}$$

When $\tau = (2\,k + 1)\,T/2$, i.e. the free electron appears at the moment of the electric field's maximum value, then it has only an oscillatory motion about the equilibrium position.

Another consequence of (2) is that the speed phase is displaced in time by $\pi/2$ as against the field phase. A consequence of this fact is that the power absorbed by the electron from the ignition field is equal to zero. Subsequently, the electron which moves without collision, doesn't take energy from the exterior field (of ignition). In this case its motion equation is:

$$m_e \frac{d^2x}{dt^2} + \nu_e m_e \frac{dx}{dt} = e\,E_m \sin \omega t \tag{7}$$

where ν_e is the frequency of electron collisions with the gas atoms.

In view of finding a relation between, the ionization field intensity, its frequency and the gas pressure, Hale finds (5) on the ground of the motion equations (2) şi (5), the following relations:

$$\left.\begin{aligned} &a)\ E_m \cdot \frac{e}{\omega \cdot m_e}\left[(t-\tau)\cos\omega\tau - \frac{1}{\omega}\sin\omega(t-\tau)\right] = \lambda_e \\ &b)\ E_m \cdot [\cos\omega\tau - \cos\omega(t-\tau)] = \left(\frac{e}{m_e} V_i\right)^{1/2} \end{aligned}\right\} \qquad (8)$$

These relations are easily obtained assuming that at the moment t the energy accumulated by the electron is sufficient for the ionization of the collided particle. The first one (a) expresses the covered space λ_e law (mean free path) in the time interval (t — τ), while the second one (b) expresses the energy acquired by the electron through variation of its speed in the same time interval.

It will be seen from (b) that for a definite ignition potential V_i, the ignition field intensity E_m, may reach a minimum, if the parenthesis value is maximum. This occurs when

$$\tau = k \cdot T/2 \text{ where } k = 0,\ 1,\ 2, \ldots$$

If $\tau = 0$, ($k = 0$), the equations (*a*) and (*b*) take the following value:

$$\left.\begin{aligned} &a')\ E_m \cdot \frac{eT}{2\pi m_e}\left(t - \frac{T}{2\pi}\sin 2\pi\cdot\frac{t}{T}\right) = \lambda_e \\ &b')\ E_m \cdot \frac{eT}{2\pi m_e}\left(1 - \cos 2\pi\frac{t}{T}\right) = \left(2\,\frac{e}{m_e}\cdot V_i\right)^{1/2} \end{aligned}\right\} \qquad (12')$$

Eliminating time from the two (*a'*) and (*b'*) equations we come to a function:

$$Em = f(\omega,\ p) \qquad (13)$$

Since $\lambda_e = g(p)$ — Drawing these functions we get the set of curves with pressure p as a parameter. The curves present a minimum for a diferite frequency. There obviously exists a frequency optimum for the ignition field. In the present work p has been:

$$p = \{15,\ 30,\ 60,\ 120\ \mu\text{Hg}\}, \qquad f\colon \{5 - 400\ \text{MHz}\}.$$

The above mentioned data form the bases of an elementary theory on the phenomenon of high frequency discharge ignition, the more complete theory makes use of statistic mechanics methods.

2. THE NECESSITY FOR COMPUTER AIDED SOLVING

To find out the function (13) time must be elimineted from the equations of the (12') system. The first step to simplify the problem is the division of the two equations (*a'*, *b'*) after which , we are led to:

$$\left(t - \frac{\sin\omega t}{\omega}\right) = a(1 - \cos\omega t) \qquad (14)$$

where a is a constant depending on λ_e, V_i, m_e şi q_e.

The solving of the equation (14), hich is a compatibility functional of the system (12) results in a function

$$t = t(\omega, a) \tag{15}$$

But it is the very hard solving of the function which sets special problems since it is an argument and argument trigonometric functions expression. It cannot be solved except by way trial and error, requiring a task which can hardly be done by a research worker. Therefore it. has been resorted to a computer.

The notation $\Delta\Sigma$ has been used ofor the equation

$$\Delta\Sigma = \omega t - a\omega - \sin \omega t + a\omega \cos \omega t \tag{16}$$

Likewise $\cos \omega t = x$; $| -1 \leqslant x \leqslant 1 |$

In order to make the solving easier and reduce the number of trials a variable step program based on the chord method has been compiled. Therefore, a special subroutine has been solved. The program is provided for the tracing of the characteristics $t = t(\omega)$, and of the $Em = f(\omega, p)$.

The result is identical with that given in literature.

Moreover, the function $t = t(\omega)$ *which is not mentioned in literature* and which represents the electron flight time between two collissions is liable to interesting interpretations on the part of physisists. Namely *it will be noted that with a definite frequency* — at constant pressure — *the flight time suddenly iincreases.* One would say that a *quasiresonance of the flight time shows up.*

3. EXISTENCE AND UNIQUENESS OF THE TRANSCEDENTAL ECQUATION SYSTEM SOLUTION FOR $t > 0$ and $\omega \leqslant \pi \frac{b}{\lambda_e} \sqrt{2}$

Lemma 1. Let us take the function f: $(0, \infty) \to \mathrm{R}$, where $\mathrm{R} = (-\infty, \infty)$, $f(x) = x - \sin x - x_a(1 - \cos x)$; $x > 0$; and x_a is a constant satisfying the condition $0 < x_a \leqslant \pi \sqrt{2}$. There exists a point $x_0 \in (0, 2\pi)$ such that $f(x_0) = 0$. The function $f(x)$ doesn't vanish any more in any point of its field of existence.

Proof

The function $f(x)$ is continuous and differentiable all over its field of existence. Thus:

$$f'(x) = 1 - \cos x - x_a \sin x$$

$$f''(x) = \sin x - x_a \cos x$$

$f'(x)$ and $f''(x)$ being also continuous and diferentiable for $x > 0$. Among the solutions of the equation $f'(x) = 0$ we find the extremum points of the function $f(x)$. If in an extremum point x_e, $f''(x\ ) < 0$ then x_e is a point of local maximum *if* $f''(x_e) > 0$ *then* x_e *is a point of local minimum.*

The inflexion points of the function $f(x)$ are to be found among the solutions of the equation $f''(x) = 0$.

Introducing the notation $y = \cos x$, the equation $f'(x) = 0$ becomes

$$1 - y - x_a \sqrt{1 - y^2} = 0$$

and represents a quadratic equation in y. This equation has the following solutions:

$$\begin{cases} y_\alpha = 1 \\ y_\beta = \dfrac{1 - x_a^2}{1 + x_a^2} \end{cases}$$

thus the equation $f'(x) = 0$ has the following solutions:

$$\begin{cases} x_{\alpha k} = 2(k + 1)\,\pi \\ x_{\beta k} = \arccos \dfrac{1 - x_a^2}{1 + x_a^2} + 2k\pi \end{cases}$$

where $k = 0, 1, 2, 3, \ldots$

$$\text{Since } 0 < \arccos \frac{1 - x_a^2}{1 + x_a^2} < \pi \text{ for } 0 < x_a \leqslant \pi\sqrt{2}$$

we have $0 < x_{\beta k} < x_{\alpha k} < x_{\beta(k+1)}$ for $k = 0, 1, 2, 3, \ldots$

After some elementary calculations we get:

$$\begin{cases} f''(x_{\alpha k}) = -\, x_a < 0 \\ f''(x_{\beta k}) = x_a > 0 \end{cases}$$

and thus $x_{\alpha k}$ are local points of maximum while $x_{\beta k}$ are local points of minimum for the function $f(x)$. The series $(x_{k\alpha})$ and $(x_{\beta k})$ are strictly increasing. Moreover, we have:

$$f(x_{\alpha k}) = x_{\alpha k} \text{ (since } \sin x_{\alpha k} = 0 \text{ and } \cos x_{\alpha k} = 1).$$

Because of

$$f(x) = x - x_a - f''(x)$$

it follows directly that:

$$f(x_{\beta k}) = x_{\beta k} - 2\,x_a, \quad \text{(since } f''(x_{\beta k}) = x_a).$$

Let us take now the function $h(x) = \arccos \dfrac{1 - x^2}{1 + x^2} - 2x$ defined on the interval $[0, \infty)$. It can be proved directly that $h(x)$ is differentiable in the $(0, \infty)$ interval and that its derivate:

$$h'(x) = 2 \cdot \left(\frac{1}{1 + x^2} - 1\right)$$

and thus $h'(x) < 0$ for $x > 0$, the function $h(x)$ being strictly decreasing in the interval $(0, \infty)$. Since $h(0). = 0$ it follows directly that $h(x) < 0$ in the $(0, \infty)$ interval. The function $h(x)$ being continuous in the interval $[0, \infty)$.

Thus:

$$\arccos \frac{1 - x^2}{1 + x^2} < 2x \text{ for } x > 0$$

and $\min h(x) = h(\pi \sqrt{2}) < -1{,}99\pi$ (after carrying out the calculations).

Finding out the difference $f(x_{\beta(k+1)}) - f_{\alpha k}(x)$ we obtain:

$$f(x_{\beta(k+1)}) - f(x_{\alpha k}) = \arccos \frac{1 - x^2}{1 + x^2} - 2x_a < 0\,;\ x_a > 0$$

und thus:

$$f(x_{\beta(k+1)}) < f(x_{\beta k})$$

Finding out $f(x_{\beta 0})$, $f(x_{\beta 1})$ and $f(x_{\alpha 0})$ we get:

$$f(x_{\beta 0}) = \text{atrc} \cos \frac{1 - x_a^2}{1 + x_a^2} - 2x_a < 0$$

$$f(x_{\beta 1}) = f(x_{\beta 0}) + 2\pi$$

$$f(x_{\alpha 0}) = 2\pi$$

The condition $f(x_{\beta 1}) < 0$ leads to the inequality:

$$\arccos \frac{1 - x_a^2}{1 + x_a^2} + 2\pi > 2x_a$$

(which might be written on this way: $h(x_a) \geqslant \min h(x_a) > -1{,}99\pi > -2\pi$) a condition obviously fulfilled for $0 < x_a \leqslant \pi \sqrt{2}$.

Therefore $f(x_{\beta k}) < 0$ for $k = 0, 1, 2, \ldots$ while $f(x_{\beta k})\ 0$ for $k = 1, 2, 3, \ldots$ and $f(x_{\beta 0}) < 0$.

Since

$$\lim_{\substack{x \to 0 \\ x > 0}} f(x) = 0$$

the function $f(x)$ vanishes only once in the $(0, \infty)$ interval. Obviously $x_0 \in (x_{\beta 0}, x_{\alpha 0})$ since $f(x_{\beta 0}) < 0$ while $f(x_{\alpha 0}) = 2\pi > 0$. In the $(x_{\beta 0}, x_{\alpha 0})$ interval, the function $f(x)$ is strictly increasing and $f'(x) < 0$; it follows that x_0 can be determined by a numerical analysis method (e.g. the „chord method“ or the „tangent method“ etc.

Thus the lemma 1 has been completely proven.

Lemma 2. Function $g(x) = \arccos \dfrac{1 - x^2}{1 + x^2} - \operatorname{arc\,tg} x$; $x \geqslant 0$ is strictly positive for $x < 0$.

Proof

$g(x)$ is a continuous function differentiable for $x > 0$ Its derivate $g'(x)$ has following espression:

$$g'(x) = \frac{1}{1 + x^2}; \quad x > 0$$

$g(x)$ being strictly increasing in the interval $(0, \infty)$. Since $g(x)$ is continuous in the interval $[0, \infty)$ and $g(o) = 0$ it follows that $g(x) > 0$ for $x > 0$ and ths the lemma has been proven.

Note

To represent graphically the function $f(x)$ we have to find the points of inflexion.

In $f''(x) = 0$ we find the points of inflexion as follows:

$$\sin x_{qk} - x_a \cdot \cos x_{qk} = 0$$

$$x_{qk} - \operatorname{arctg} x_a + k\pi; \; f''(x_{qk}) = 0; \; K = 0, 1, 2, \ldots$$

The series (x_{qk}) is strictly increasing and the difference between two of its consecutive terms $(x_{q(k+1)} - x_{qk})$ equals π.

$0 < x_{q0} = \operatorname{arctg} x_a < x_{\beta 0} = \operatorname{arc} \cos \frac{1 - x_a^2}{1 + x_a^2}$ on the ground of lemma 2.

$$x_{q1} = \pi + x_{q0} < \frac{3\pi}{2}, \qquad \text{since } 0 < x_{q0} < \frac{\pi}{2}$$

and $x_{q1} < x_{\alpha 0} = 2\pi$

Consider $h^*(x_a) = x_{\beta 0} - x_{q1} = \operatorname{arc} \cos \frac{1 - x_a^2}{1 + x_a^2} - \operatorname{arc} \operatorname{tg} x_a - \pi = g(x_a) - \pi.$

Since $g(x_a) < \lim\limits_{x \to \infty} g(x) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ it follows directly that

$$h^*(x) < \lim_{x \to \infty} g(x) - \pi = -\frac{\pi}{2}$$

and thus $x_{\beta 0} < x_{q1}$ for $x > 0$.

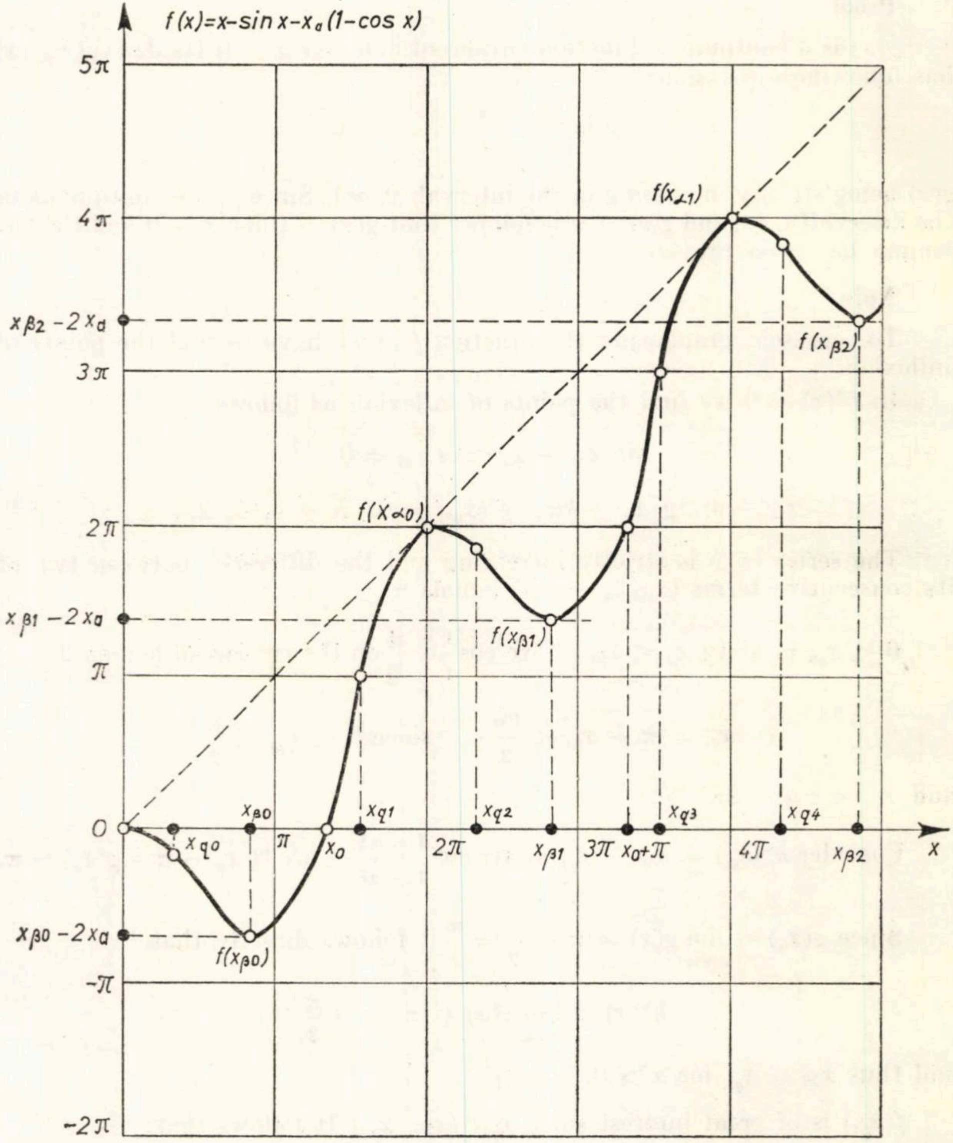
$f(x_{q1})$ is of great interest since $x_{q1} \in (x_{\beta 0}, x_{\alpha 0})$ It follows that:

$$f(x_{q1}) = x_{q1} - x_a = \pi + \operatorname{arctg} x_a - x_a$$

and $f(x_{q1}) > 0$ because of $0 < x_a \leqslant \pi\sqrt{2}$

As a conclusion point $x_0 \in (x_{\beta q}, x_{q1})$ since $f(x_{\beta 0}) < 0$ and $f(x_{21}) > 0$ and $(x_{\beta 0}, x_{q1}) \subseteq (x_{\beta 0}, x_{\alpha 0})$.

Taking into account lemma 1, lemma 2 and the previous observations the function $f(x)$ might be reprsented graphically as fig. 1



Fig, 1

Theorem. Consider the system of tramscendental equations:

$$\left.\begin{aligned} E\cdot q_e\cdot\frac{1}{\omega}\left(t-\frac{\sin(\omega t)}{\omega}\right) &= \lambda_e \\ E\cdot q_e\cdot\frac{1}{\omega}\,(1-\cos(\omega t) &= b \end{aligned}\right\} \tag{17}$$

where λ_e, b, E, q_e, ω are strictly positive constant quantities while $t > 0$ is an independent variable.

$$If\ \omega \leqslant \pi \frac{b}{\lambda_e} \cdot \sqrt{2}$$

then the system (17) has a unique solution $t_0 > 0$.

(Note — The system (17) is equivalent to the system (12′) where the following denotations have been made:

$$q_e = \frac{e}{m_e}; \quad \omega = \frac{2\pi}{T} \qquad \text{și} \sqrt{2 \frac{e}{m_e V_i}} = b$$

PROOF OF THE THEOREM

The right-hand side of the last equation of the system is non-zero regardless of $t > 0$. Dividing the 2nd equation by the 1 st equation (the division is performed term by term) it follows that:

$$\frac{1 - \cos(\omega t)}{t - \dfrac{\sin(\omega t)}{\omega}} = \frac{b}{\lambda_e} \tag{18}$$

When denoting:

$$\left.\begin{aligned} a &= \frac{\lambda_e}{b} > 0 \\ x_a &= a \cdot \omega > 0 \end{aligned}\right\} \tag{19}$$

the equation (18) becomes:

$$x_a(1 - \cos(\omega)) = \mathrm{t} - \sin(\omega t). \tag{20}$$

After the change of variable

$$x = \omega t \text{ (Note: } x > 0 \text{ since } \omega > 0 \text{ and } t > 0) \tag{21}$$

the equation (20) may be written in this form:

$$x - \sin x - x_a(1 - \cos x) = 0 \tag{22}$$

where:

$$0 < x_a = a \cdot \omega \leqslant \frac{\lambda_e}{b} \cdot \pi \cdot \frac{b}{\lambda_e} \cdot \sqrt{2} = \pi\sqrt{2}.$$

Denoting the right-hand expression of the equation (22) by $f(x)$ it follows that the equation (22) may be written as follows:

$$f(x) = 0 \tag{23}$$

where the function $f(x)$ is identical with the one met in lemma 1.

According to lemma 1 there exists a unique $x_0 > 0$, so that

$$f(x_0) = 0.$$

In this case we determine t_0,

$$t_0 = \frac{x_0}{\omega} \; (t_0 > 0 \text{ since } x_0 > 0), \tag{24}$$

which obviously is a solution of the system (17), moreover the unique solution for $t > 0$, since any solution t of the system (17) uniquely determines the solution „x“ (in the form $x = \omega t$) of the equation (23) and reciprocally.

Thus, the theorem is completely proven.

4. THE ALGORITHM OF THE COMPUTER-AIDED SOLVING OF THE TRANSCENDENTAL EQUATIONS SYSTEMS

It results from the above mentioned that the solution t_0 of the transcendental equations system can be obtained by solving the equation $f(x) = 0$ where $x = \omega t$ and $f(x) = x - \sin x - x_a(1 - \cos x)$ and $x_a = \frac{\lambda_e}{b}\,\omega$. After finding the solution x_0 for which $f(x_0) = 0$ we come to the solution $t_0 = \frac{x_0}{\omega}$ of the transcendetal equations system.

It has been shown that $x_0 \in (\arccos\frac{1 - x_a^2}{1 + x_a^2},\ \pi + \operatorname{arctg} x_a)$ and that in the previous interval $f'(x) > 0$.

It follows that in this space x_0 can be determined by way of the „chord method“, using the subroutine PUNCT (POINT).

Considering innitially verified the following conditions:

$$q_e > 0; \quad \lambda_e > 0; \quad b > 0; \quad E > 0; \quad \omega_{min} > 0; \quad \omega_{max} \leqslant \frac{b}{\lambda_e}\pi\sqrt{2}$$

the logical schema of the algorithm for $t_0 = t_0(\omega)$ values, when $\omega_{min} \leqslant \omega \leqslant \omega_{max}$ may be set up; this logical scheme is shown in fig. 5, annex II.

In this flow chart the following innitial data have been presumed:

$$\begin{array}{lll}
\{q_i\} & i = 1, 2, \ldots, n_q & \text{where } n_q \leqslant 5; \\
\{V_j\} & j = 1, 2, \ldots, n_v & \text{where } n_v \leqslant 5; \\
\{\lambda_k\} & k = 1, 2, \ldots, n_\lambda & \text{where } n_\lambda \leqslant 5; \\
\{\omega_m\} & m = 1, 2, \ldots, n_\omega & \text{where } \begin{cases} n_\omega \leqslant 50 \\ \omega_1 = \omega_{min} \\ \omega_{n_\omega} = \omega_{max} \end{cases}
\end{array}$$

which during a single running of the program leads to the effectuation of the computations for at most 5 values of $\lambda_e \in \{\lambda_k\}$, 5 values of $q_e \in \{q_i\}$ and 25 values of b (values found by way of the relation $b_{ij} = \sqrt{2q_i V_j}$. For each triplet (q_i, b_{ij}, λ_k) the computation is performed for mostly 50 distinct values of $\omega \in \{\omega_m\}$. Therefore, a two-pair set of values q_i, V_j, λ_k, ω_m asks for a unique determination of q_e, b, λ_e and ω so that

$$q_e = q_i ; \quad b = \sqrt{2q_i V_j} ; \quad \lambda_e = \lambda_k ; \quad \omega_m = \omega$$

(the indices i, j, k, m being fixed up for the two — pair set of values taken into account). The computations start everytime with the two — pair set of values q_1, V_1, λ_1, ω_1 and are carried out for all the $(n_q \cdot n_v \cdot n \cdot n_\omega)$ 2-pair sets of values q_j, V_k, λ_m, ω_m so that the indices i, j, k, m vary in a reversed lexicographical order (i.e. m varies more rapidly then k, etc.).

The program has been compiled in the FORTRAN language and has been run on the IBM/360 computer from the computing Centre of the Bucharest University, for the following series of values:

$q_1 = 0{,}1759 \cdot 10^{12}$ Coulombs; $n_q = 1$;
$V_1 = 0,\ 157 \cdot 10^2$ Voltsλ $n_v = 1$;
$\lambda_1 = 0{,}338 \cdot 10^{-2}$ metres;
$\lambda_2 = 0{,}169 \cdot 10^{-2}$ metres;
$\lambda_3 = 0{,}845 \cdot 10^{-3}$ metres;
$\lambda_4 = 0{,}4225 \cdot 10^{-3}$ metres; $n_\lambda = 4$;
$\omega_1 = 2\pi \cdot 5 \cdot 10^6$ rad/sec
$\omega_m = (2m - 1)\,\omega_1$; $m = 2, 3, 4, \ldots, 40$; $n_\omega = 40$.

It can easily be seen that $q_e = q_1$, $b = \sqrt{2q_1 V_1}$; $\omega_{min} = \omega_1$ and $\omega_{max} = 79 \cdot \omega_1 < 2\pi \cdot 4 \cdot 10^8 \frac{\text{rad}}{\text{sec}}$. The condition $\omega_{max} \leqslant \frac{b}{\lambda_e} \pi \sqrt{2}$ is fulfilled since

$$2\pi \cdot 4 \cdot 10^8 \leqslant \pi \cdot \frac{0{,}236 \cdot 10^7}{0{,}338 \cdot 10^{-2}} \cdot \sqrt{2} = 9{,}88 \cdot \pi \cdot 10^8$$

and because q_e, b, λ_e, ω are strictly positive, the solution $t_0 = t_0(\omega)$ of the trascendent equations system is unique and $t_0 > 0$.
$(n_q \times n_v \times n_\lambda \times n_\omega = 1 \times 1 \times 4 \times 40 = 160$ values have been obtained for $t_0 = t_0\,(q_i, V_j, \lambda_k, \omega_m)$ in a total execution time of 2 minutes 40 seconds).

It is noteworthy that to depends only on the $\frac{\lambda_e}{b}$ ratio and not on q_e and E when $q_e > 0$ and $E > 0$ since

$$a = \frac{\lambda_e}{b} = \frac{\dfrac{e}{E \cdot q_e}}{\dfrac{b}{E \cdot q_e}} = \frac{\lambda^*}{b^*}$$

where $\lambda^* = \frac{e}{E \cdot q_e}$ and $b^* = \frac{b}{E \cdot q_e}$, the transcendent equations system thus becoming:

$$\begin{cases} \frac{1}{\omega}\left(t - \frac{\sin(\omega t)}{\omega}\right) = \lambda^* \\ \frac{1}{\omega}(1 - \cos(\omega t)) = b^* \end{cases}$$

Thus, when knowing t_0, ω, λ_e and q_e one could obtain E from the first equation of the system in the form:

$$E = E(t_0, \omega, \lambda_e, q_e) = \frac{\omega \cdot \lambda_e}{q_e\left(t_0 - \frac{1}{\omega}\sin(\omega t_0)\right)}$$

and therefore no initiation of E in the computation has been necessary.

The main program as results from fig. 5 uses two subroutines PUNCT and GRAFIC whose description is briefly done in annexes I and II.

5. RESULTS OBTAINED BY COMPUTER

Following the computation program, the data obtained by computer are:

Table nr. 1 wherein the values for $\lambda_e = 0{,}845.10^{-2}$m only are presented

Chart of the function $t = t(\omega, \lambda_e)$

TABLE WITH THE MINIMUM AND MAXIMUM VALUES OF THE FUNCTION

FUNCTION (Simbol of representation)	VALUE		ARGUMENT		TABLE STEP	
	Minimum	Maximum	Minumum	Maximum	On the abscisa	On the ordonate
*	0.1713E-08	0.4098E-08	0.3141E 08	0.2544E 10	0.6283E 08	0.4770E-10
.	0.1358E-08	0.2045E-08	0.3141E 08	0.2544E 10	0.6283E 08	0.1375E-10
,	0.7871E-09	0.1027E-08	0.3141E 08	0.2544E 10	0.6283E 08	0.4798E-11
+	0.3959E-09	0.5115E-09	0.3141E 08	0.2544E 10	0.6283E 08	0.2312E-11

Table coresponding to the arguments and values of the functions from fig. 2

INDEX	X	FUNCTOIN*(X)	FUNCTION. (X)	FUNCTION, (X)	FUNCTION+(X)
1	0.3141E 08	0.3144E-08	0.1692E-08	0.8015E-09	0.4005E-09
2	0.9424E 08	0.3102E-08	0.1578E-08	0.8011E-09	0.4011E-09
3	0.1571E 09	0.3096E-09	0.1572E-08	0.7707E-09	0.4016E-09
4	0.2200E 09	0.3907E-08	0.1568E-08	0,8008E-08	0.4023E-09
5	0.2828E 09	0.4098E-08	0.1564E-08	0.8008E-09	0.4028E-09
6	0.3456E 09	0.4003E-08	0.1550E-08	0.7966E-09	0.4023E-09
7	0.4084E 09	0.3889E-08	0.1949E-08	0.7949E-09	0.4017E-09
8	0.4712E 09	0.3833E-08	0.1935E-08	0.7932E-09	0.3999E-09
9	0.5341E 09	0.3717E-08	0.2045E-08	0.7925E-09	0.3995E-09
10	0.5969E 09	0.3621E-08	0.2025E-08	0.7902E-09	0.3991E-09
11	0.6597E 09	0.3525E-08	0.2001E-08	0.7178E-09	0.3988E-09
12	0.7225E 09	0.3430E-08	0.1977E-08	0.8494E-09	0.3985E-09
13	0.7854E 09	0.333E5-08	0.1948E-08	0.9118E-09	0.3982E-09
14	0.8482E 09	0.2339E-08	0.1922E-08	0.9735E-09	0.3980E-09
15	0.9110E 09	0.3144E-08	0.1908E-08	0.9694E-09	0.3976E-09
16	0.9739E 09	0.3048E-08	0.1894E-08	0.1027E-08	0.3970E-09
17	0.1037E 10	0.2953E-08	0.1866E-08	0.1022E-08	0.3966E-09
18	0.1099E 10	0.2858E-08	0.1853E-08	0.1017E-08	0.3964E-09
19	0.1162E 10	0.2762E-08	0.1826E-08	0.1011E-08	0.3962E-09
20	0.1225E 10	0.2715E-08	0.1798E-08	0.1008E-08	0.3960E-09
21	0.1288E 10	0.2623E-08	0.1770E-08	0.1003E-08	0.3959E-09
22	0.1351E 10	0.2572E-08	0.1743E-08	0.9982E-09	0.3959E-09
23	0.1414E 10	0.2476E-08	0.1736E-08	0.9885E-09	0.4259E-09
24	0.1476E 10	0.2428E-08	0.1702E-08	0.9838E-09	0.4560E-09
25	0.1539E 10	0.2380E-08	0.1678E-08	0.9790E-09	0.4548E-09
26	0.1602E 10	0.2333E-08	0.1647-08E	0.9646E-09	0.4861E-09
27	0.1665E 10	0.2238E-08	0.1625E-08	0.9625E-09	0.4848E-09
28	0.1728E 10	0.2190E-08	0.1606E-08	0.9598E-09	0.4841E-09
29	0.1791E 10	0.2142E-08	0.1578E-08	0.9632E-09	0.4837E-09
30	0.1853E 10	0.2094E-08	0.1552E-08	0.9590E-09	0.5115E-09
31	0.1916E 10	0.2047E-08	0.1536E-08	0.9502E-09	0.5108E-09
32	0.1979E 10	0.1999E-08	0.9454E-09	0.9454E-09	0.5101E-09
33	0.2042E 10	0.1957E-08	0.1495E-08	0.9406E-09	0.5098E-09
34	0.2105E 10	0.1931E-08	0.1471E-08	0.9358E-09	0.5093E-09
35	0.2168E 10	0.1904E-08	0.1450E-08	0.9262E-09	0.5079E-09
36	0.2230E 10	0.1863E-08	0.1432E-08	0.9150E-09	0.5058E-09
37	0.2293E 10	0.1818E-08	0.1413E-08	0.9118E-09	0.5045E-09
38	0.2356E 10	0.1760E-08	0.1399E-08	0.9050E-09	0.5028E-09
39	0.2419E 10	0.1743E-08	0.1369E-08	0.9022E-09	0.5016E-09
40	0.2481E 10	0.1713E-08	0.1358E-08	0.8974E-09	0.4999E-09

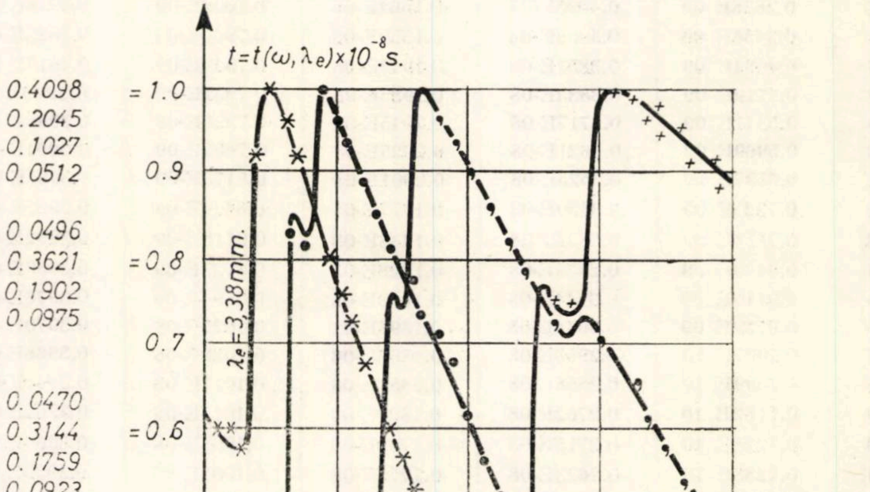


Fig. 2. **The chart of the function** $t = t(\omega, \lambda_e)$

Table corresopnding to the argument and values the functions from fig. nr.3

INDEX	X	FUNCTION*(X)	FUNCTION. (X)	FUNCTION, (X)	FUNCTION+(X)
1	0.3142E 08	0.8116E 05	0.3401E 06	0.1364E 07	0.5476E 07
2	0.9425E 08	0.1563E 05	0.7036E 05	0.4537E 06	0.1813E 07
3	0.1571E 09	0.9805E 04	0.3722E 05	0.1954E 06	0.1089E 08
4	0.2199E 09	0.7330E 04	0.2704E 05	0.1054E 06	0.7783E 06
5	0.2827E 09	0.6246E 04	0.2151E 05	0.8242E 05	0.6059E 06
6	0.3456E 09	0.5603E 04	0.1802E 05	0.6792E 05	0.3076E 06
7	0.4084E 09	0.5256E 04	0.1573E 05	0.5796E 05	0.2267E 06
8	0.4712E 09	0.5091E 04	0.1413E 05	0.5073E 05	0.1970E 06
9	0.5341E 09	0.5052E 04	0.1295E 05	0.4527E 05	0.1743E 06
10	0.5969E 09	0.5104E 04	0.1209E 05	0.4101E 05	0.1564E 06
11	0.6597E 09	0.5227E 04	0.1146E 05	0.3750E 05	0.1420E 06
12	0.7226E 09	0.5407E 04	0.1099E 05	0.3474E 05	0.1302E 06
13	0.7854E 09	0.5636E 04	0.1064E 05	0.3246E 05	0.1203E 06
14	0.8482E 09	0.5907E 04	0.1040E 05	0,3056E 05	0.1119E 06
15	0.9111E 09	0.6216E 07	0.1024E 05	0,2896E 05	0.1047E 06
16	0.9739E 09	0.6560E 04	0.1014E 05	0.2760E 05	0.9845E 05
17	0.1037E 10	0.6937E 04	0.1010E 05	0.2642E 05	0.9300E 05
18	0.1100E 10	0.7346E 04	0.1011E 05	0,2543E 05	0.8821E 05
19	0.1162E 10	0.7783E 04	0.1017E 05	0.2457E 05	0.8396E 06
20	0.1225E 10	0.8250E 04	0.1026E 05	0.2384E 05	0.8018E 05

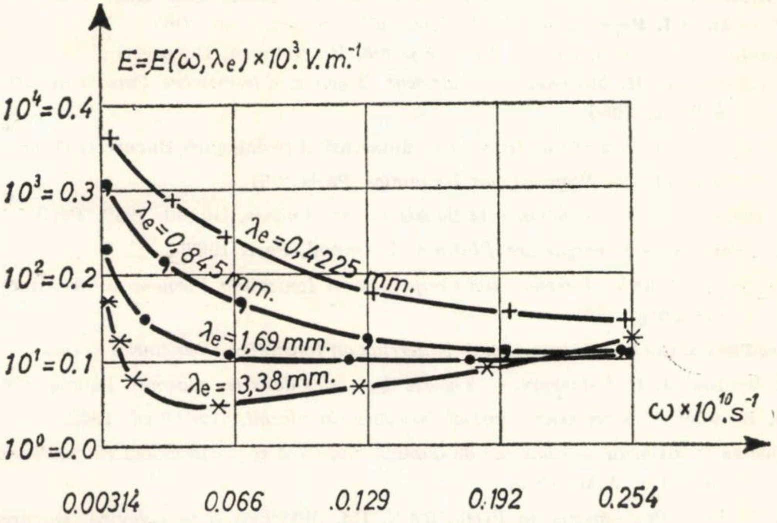


Fig. 3. Chart of the function $E = E(\omega, \lambda_e)$

CONCLUSIONS

The present work is built round the idea of experimenting on the computer some physical hypotheses and comes to conclusions without performing any further physical experiments, which are to be dons only the computed results run counter to expectancies or in order to make up convictions.

The existence of an ignition potential optimum related to frequency can be observed. It shifts towards greater frequencies along with increasing pressure. Likewise the optimum zone grows less clear along with the increasing pressure.

The existence of an optimum may present a criterion for the apparatus manufactures and is at the same time an indication that the efficiency of the installation may be better. This is of great importance for the gas discharges chemistres. Unfortunateb, the problem of gas mixture discharges can hardly be treated theoretically by and the present work is a recommendation to resort to the computer.

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Annex I

Diseription of the PUNCT subroutine

1. *Denomination :* SUBROUTINE PUNCT (OMEGA, X1, XA, TOL, TI)

2. *Object :* The determination of the solutions of the equation $f(x) = 0$ where $f(x) = x - \sin x - x_a(1 - \cos x)$;

$$x_a = XA.$$

$$x(X1, 2\pi);\ f'(x) > 0;$$

$$X1 = \text{arc}\cos\frac{1 - x_a^2}{1 + x_a^2}$$

The approximated solution of the equation $f(x) = 0$ ist the first value x_0^* for which $f(x_0^*) \leqslant$ TOL Consider initially $dx = \frac{X1}{10}$ and search for a space in the $[y, z]$ from where $z = y + dx$ and $y = k \cdot X1$, k being a natural number, such that $f(y) \cdot f(z) < 0$; $f(y) < 0$ and $f(z) > 0$.

In the $[y, z]$ space obtained in this way we consider $x_0^* = w$ and therefore $t_i = \frac{x_0^*}{\text{OMEGA}}$; *if* $|f(w)| >$ TOL and $f(w) > 0$ we assume $z = w$ and resume the procedure, whereas if $|f(w)| >$ TOL and $f(w) < 0$ we assume $y = w$ and resume the procedure for the new space $[y, z]$ thus determined until we find $|f(w)| \leqslant$ TOL.

Since ω = OMEGA it follows that $t_i = t_0^*(\omega)$ is the approximated solution of the transcendent equations system.

3. *Input arguments*

OMEGA — Real variable. Its value $\omega > 0$ is made sure by the program calling the subroutine PUNCT.

X1 — Real variable. Its value:

$$x = \text{arc}\cos\frac{1 - x_a^2}{1 + x_a^2} > 0$$

is ensured by the program calling the subroutine PUNCT.

XA — Real variable. Its value:

$$x_a > 0 \text{ (where } x_a = \frac{\omega \cdot \lambda_e}{b} \leqslant \pi\sqrt{2})$$

ensured by the programme calling the subroutine PUNCT.

TOL — Real variable. Its value $\varepsilon > 0$.

4. *Output arguments* TI — Real variable, conveying the value $t_1 = t_0^*(\omega)$ computed so that $|f(t_i)| \leqslant \varepsilon$

5. *Necessary subroutines :* None

6. *Usage statement :* CALL PUNCT (A, B, C, D, E) where A, B, C, D, E are real variables, not not indexed, simple precision.

7. *Logical scheme :* presented in fig.4.

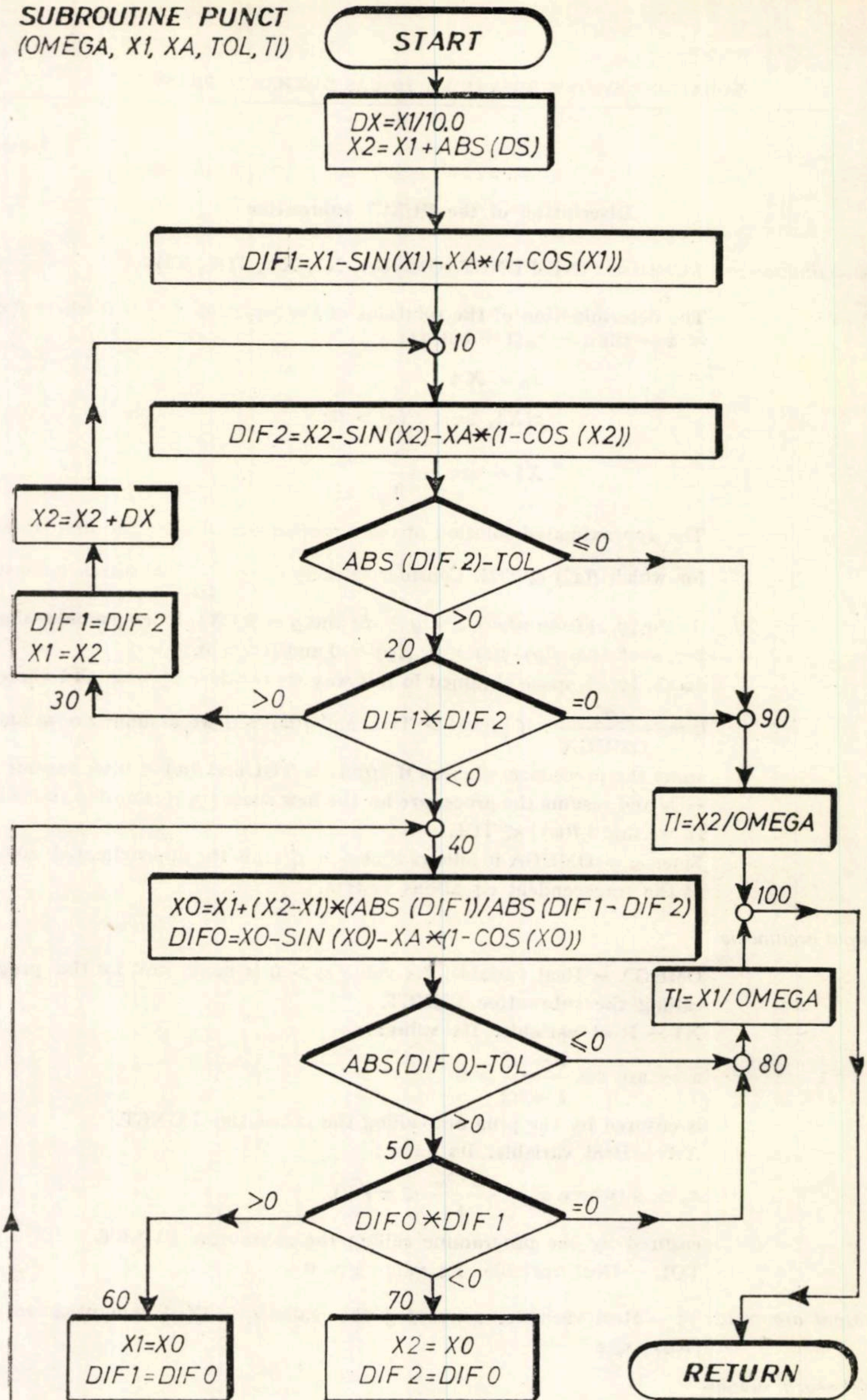

SUBROUTINE PUNCT
(OMEGA, X1, XA, TOL, TI)
START
DX=X1/10.0
X2=X1+ABS(DS)
DIF1=X1-SIN(X1)-XA*(1-COS(X1))
10
DIF2=X2-SIN(X2)-XA*(1-COS (X2))
X2=X2+DX
ABS (DIF 2)-TOL
≤0
>0
DIF1=DIF2
X1=X2
20
30
>0
DIF1*DIF 2
=0
90
<0
TI=X2/OMEGA
40
XO=X1+(X2-X1)*(ABS (DIF1)/ABS (DIF 1- DIF 2)
DIFO=XO-SIN (XO)-XA*(1-COS (XO))
100
TI=X1/OMEGA
ABS(DIF O)-TOL
≤0
80
>0
50
>0
DIFO*DIF 1
=0
<0
60
70
X1=XO
DIF1=DIF O
X2=XO
DIF 2=DIF O
RETURN

Fig. 4

Annex II

Logical diagram and program in FORTRAN

În fig. 5 the logical scheme of the principal programme written in the FORTRAN language is presented. In the logical scheme and the program the following names for the variables of the solving algorithm of the problem are used:

LAMBDA (K) $\Rightarrow \lambda_k$; NL $\Rightarrow n_\lambda$
V(J) $\Rightarrow V_j$; NV $\Rightarrow n_v$
OMEGA I (M) $\Rightarrow \omega_m$; NO $\Rightarrow n_\omega$
Q(I) $\Rightarrow q_i$; NQ $\Rightarrow n_q$
VI $\Rightarrow V_i$;
LP $\Rightarrow \lambda_e$
QP $\Rightarrow q_e$
A $\Rightarrow a = \dfrac{\lambda_e}{b}$
B $\Rightarrow b = \sqrt{2 q_e V_i}$
Y $\Rightarrow \omega$
X1 $\Rightarrow x_{\beta 0} = \arccos \dfrac{1 - x_a^2}{1 + x_a^2}$
XA $\Rightarrow x_a = a.\omega$
TOL $\Rightarrow \varepsilon > 0$ (tolerance such that $| f(x_0^*) | \leqslant \varepsilon$)
TI $\Rightarrow t_0^* = \dfrac{x_0^*}{\omega}$
E1 (IL, I) $\Rightarrow E(\lambda_k, \omega_m)$
TIMP (IL, I) $\Rightarrow t_0^*(\lambda_k, \omega_m)$

Subsequently the main program and the PUNCT and GRAFIC subroutines written in the FOTRAN language are given.

The subroutine GRAFIC is a subroutine for the automatic racing of real functions of real variable (at most 5 functions) defined in the same field of definition. On the ordinate and absscisa we have linear scales; in addition to this the automatic norming of the scale on the ordinate is performed separately for each function; it follows that the ordinate scales differ from each other for different functions.

The subroutine GRAFIC was conceived and carried out completely by the authors themselves. To ease the interpreting of results, the tables of the maximum and minimum values of the functions as well as the value of 5 signifient figures of the functions in the points of the graphic are presented.

By means of the subroutine GRAFIC $t = t(\omega, \lambda_e)$ (fig. 2) and $E = E$ (ω, λ_e) (fig. 3) as well as the greatest part of obtained results have been pointed.

PROGRAMUL PRINCIPAL

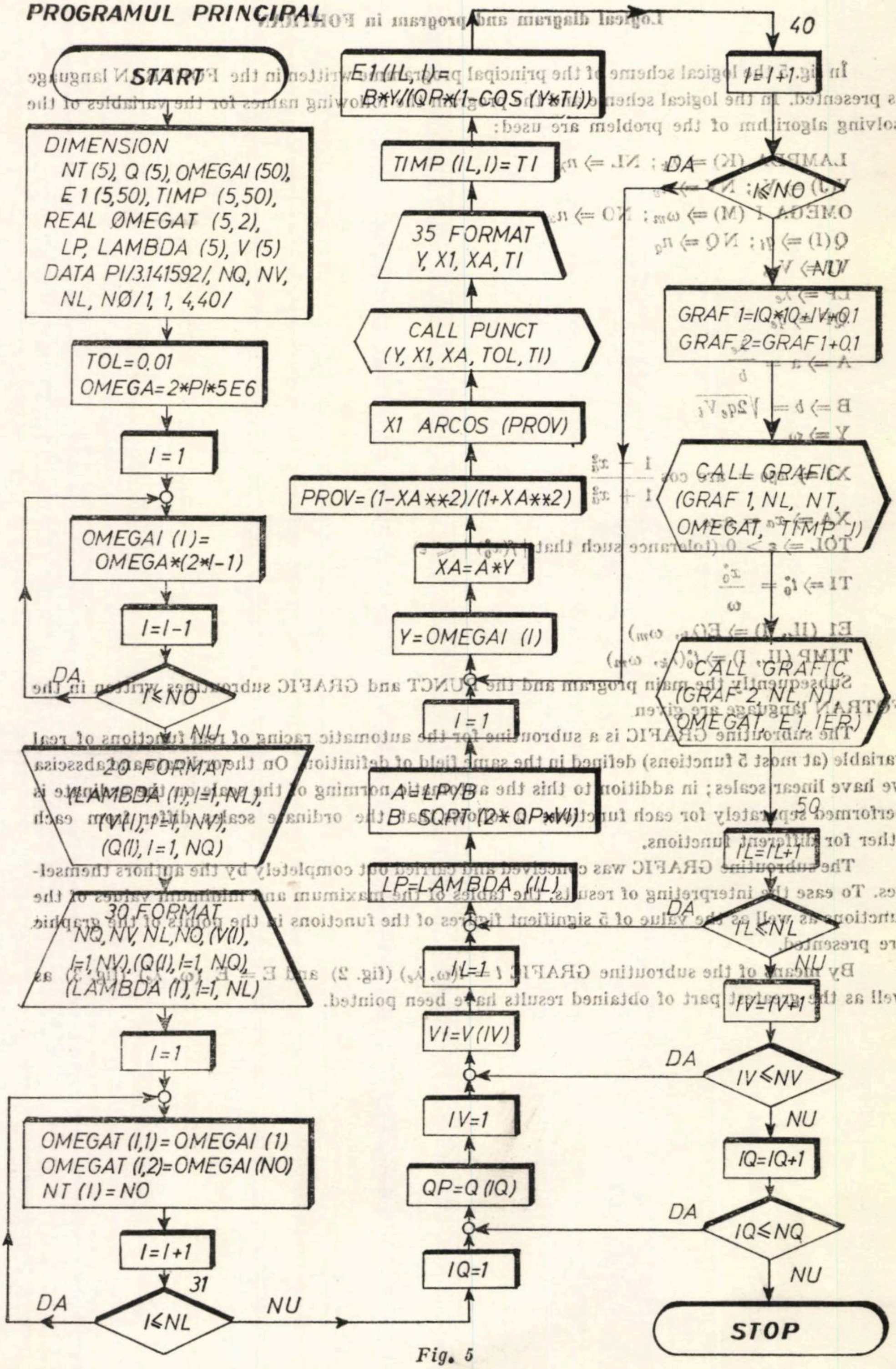
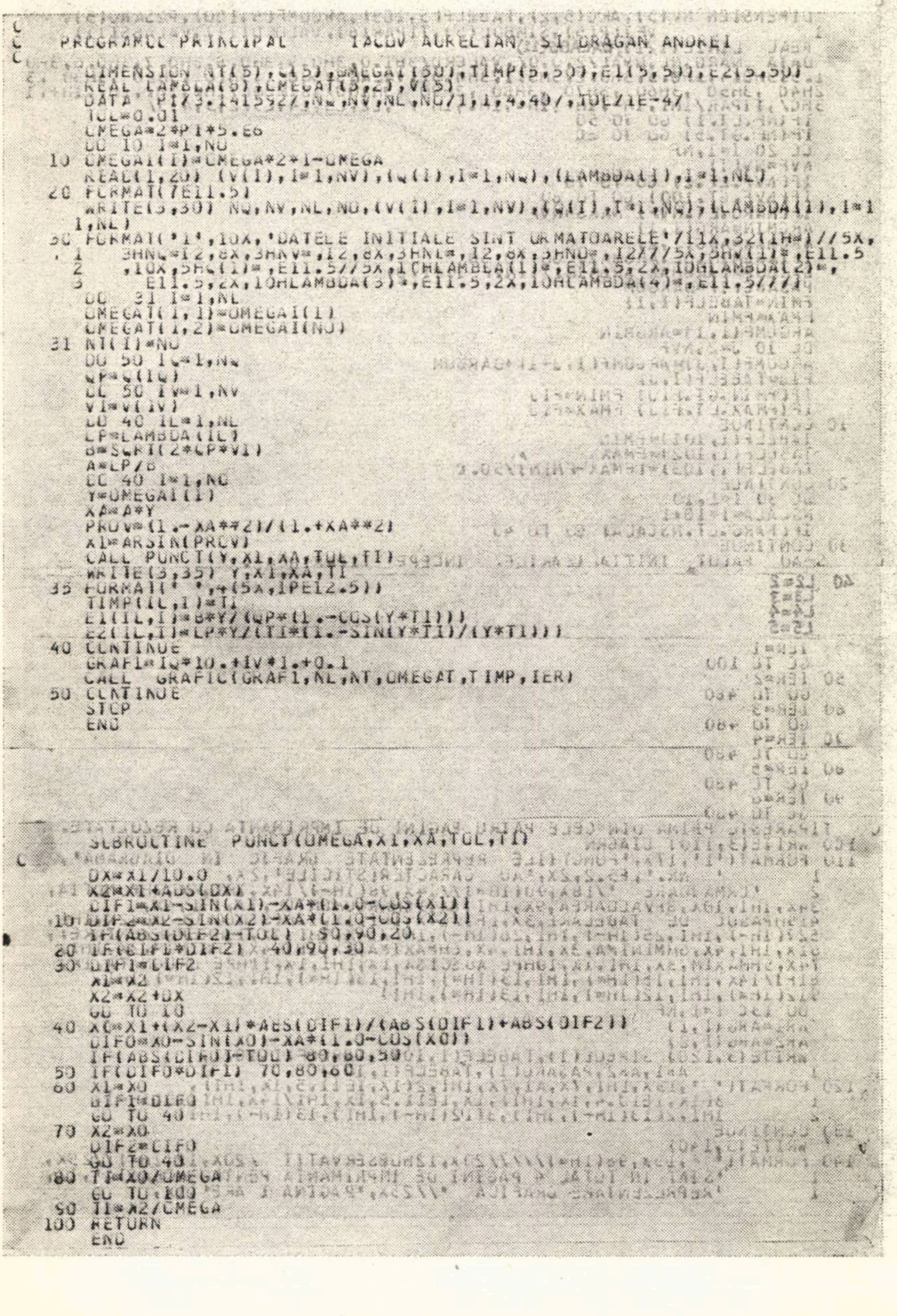
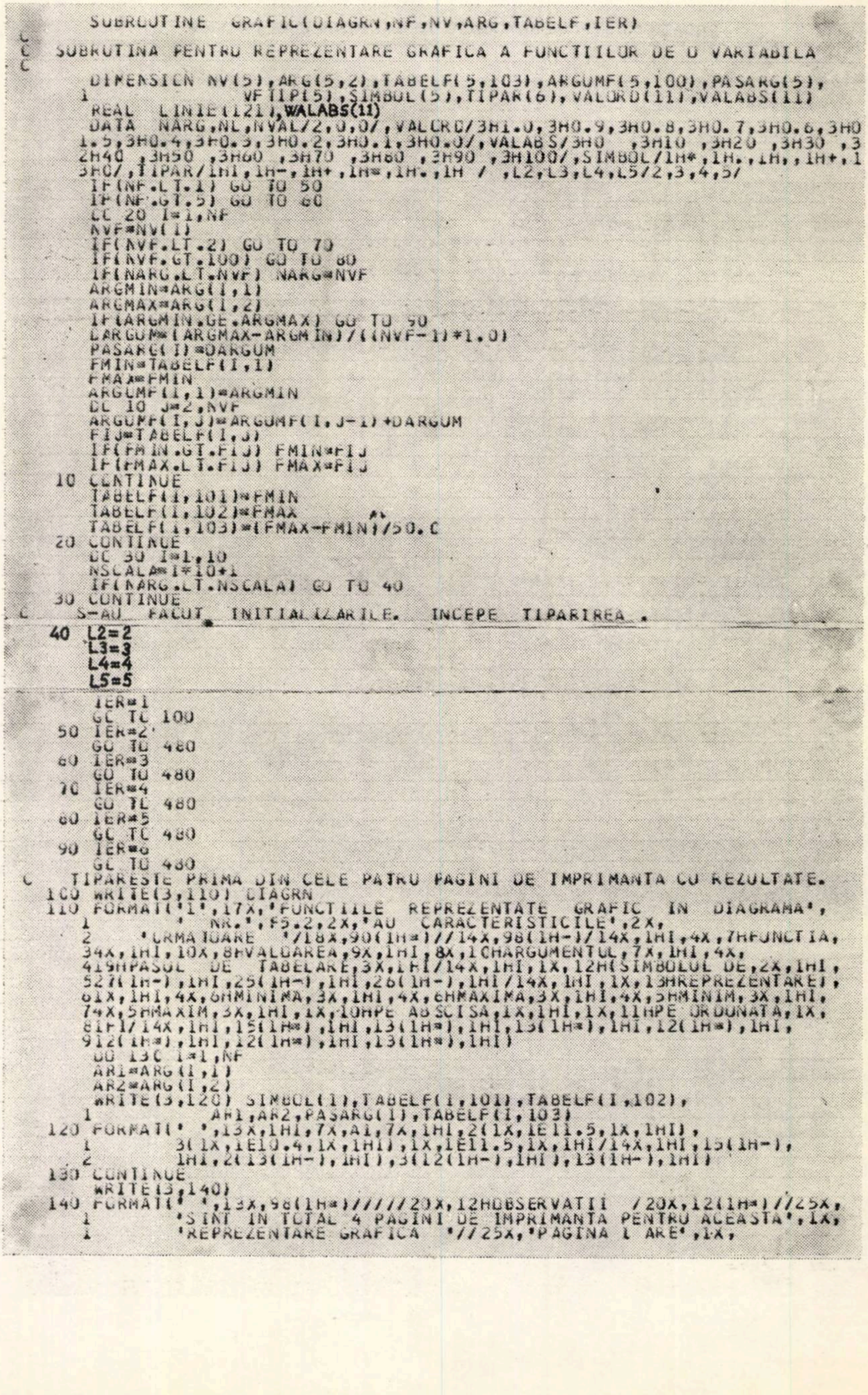

START
DIMENSION
NT (5), Q (5), OMEGAI (50),
E 1 (5,50), TIMP (5,50),
REAL ØMEGAT (5, 2),
LP, LAMBDA (5), V (5)
DATA PI/3.141592/, NQ, NV,
NL, NØ/1, 1, 4,40/
TOL=0.01
OMEGA=2*PI*5 E6
I = 1
OMEGAI (I)=
OMEGA*(2*I-1)
I=I-1
DA
I≤NO
NU
20 FORMAT
(LAMBDA (I), I=1, NL),
(V(I), I=1, NV),
(Q(I), I=1, NQ)
30 FORMAT
NQ, NV, NL, NO, (V(I),
I=1, NV), (Q(I), I=1, NQ),
(LAMBDA (I), I=1, NL)
I = 1
OMEGAT (I,1) = OMEGAI (1)
OMEGAT (I,2)=OMEGAI (NO)
NT (I) = NO
I = I +1
31
DA
I≤NL
NU
E1 (IL, I)=
B*Y/(QP*(1-COS (Y*TI))
TIMP (IL, I)= T I
35 FORMAT
Y, X1, XA, TI
CALL PUNCT
(Y, X1, XA, TOL, TI)
X1 ARCOS (PROV)
PROV= (1-XA**2)/(1+XA**2)
XA=A*Y
Y=OMEGAI (I)
I = 1
A=LP/B
B SQRT(2*QP*VI)
LP=LAMBDA (IL)
IL=1
VI=V(IV)
IV=1
QP=Q (IQ)
IQ=1
40
I=I+1
DA
I≤NO
NU
GRAF 1=IQ*10+IV*0.1
GRAF 2=GRAF 1+0.1
CALL GRAFIC
(GRAF 1, NL, NT,
OMEGAT, TIMP, J)
CALL GRAFIC
(GRAF 2, NL, NT,
OMEGAT, E1, IER)
50
IL=IL+1
DA
IL≤NL
NU
IV=IV+1
DA
IV≤NV
NU
IQ=IQ+1
DA
IQ≤NQ
NU
STOP

Fig. 5

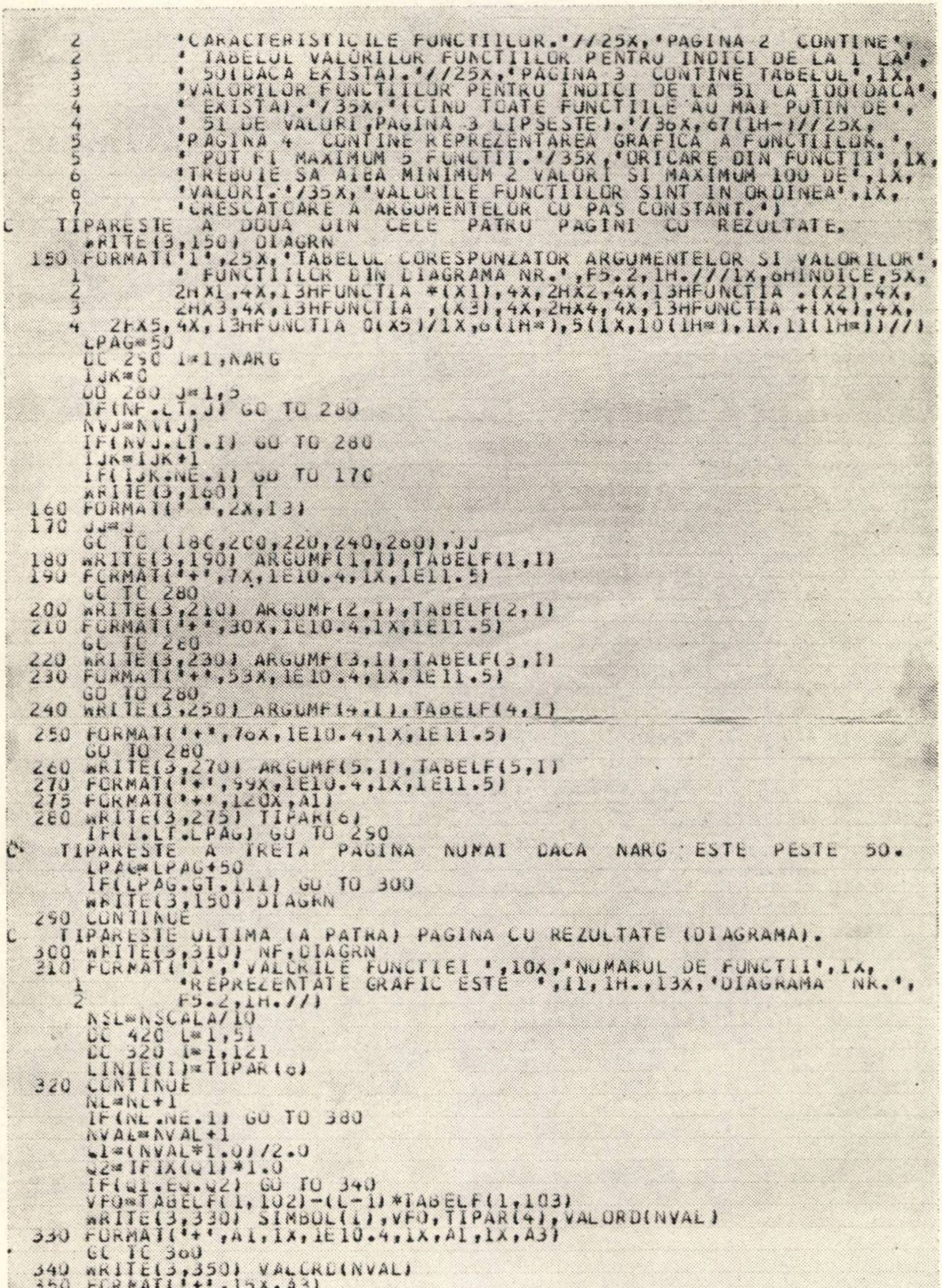


```
C
C   PROGRAMUL PRINCIPAL     IACOV AURELIAN SI DRAGAN ANDREI
C
      DIMENSION NT(5),Q(5),OMEGAI(50),TIMP(5,50),E1(5,50),E2(5,50)
      REAL LAMBDA(5),OMEGAT(5,2),V(5)
      DATA  PI/3.141592/,NQ,NV,NL,NO/1,1,4,40/,TOL/1E-4/
      TOL=0.01
      OMEGA=2*PI*5.E6
      DO 10 I=1,NO
   10 OMEGAI(I)=OMEGA*2*I-OMEGA
      READ(1,20) (V(I),I=1,NV),(Q(I),I=1,NQ),(LAMBDA(I),I=1,NL)
   20 FORMAT(7E11.5)
      WRITE(3,30) NQ,NV,NL,NO,(V(I),I=1,NV),(Q(I),I=1,NQ),(LAMBDA(I),I=1
     1,NL)
   30 FORMAT('1',10X,'DATELE INITIALE SINT URMATOARELE'/11X,32(1H=)//5X,
    1    3HNQ=I2,8X,3HNV=,I2,8X,3HNL=,I2,8X,3HNO=,I2//5X,5HV(1)=,E11.5
    2    ,10X,5HQ(1)=,E11.5//5X,10HLAMBDA(1)=,E11.5,2X,10HLAMBDA(2)=,
    3      E11.5,2X,10HLAMBDA(3)=,E11.5,2X,10HLAMBDA(4)=,E11.5///)
      DO  31 I=1,NL
      OMEGAT(I,1)=OMEGAI(1)
      OMEGAT(I,2)=OMEGAI(NO)
   31 NT(I)=NO
      DO 50 IQ=1,NQ
      QP=Q(IQ)
      DO 50 IV=1,NV
      VI=V(IV)
      DO 40 IL=1,NL
      LP=LAMBDA(IL)
      B=SQRT(2*QP*VI)
      A=LP/B
      DO 40 I=1,NO
      Y=OMEGAI(I)
      XA=A*Y
      PROV=(1.-XA**2)/(1.+XA**2)
      X1=ARSIN(PROV)
      CALL PUNCT(Y,X1,XA,TOL,TI)
      WRITE(3,35) Y,X1,XA,TI
   35 FORMAT(' ',4(5X,1PE12.5))
      TIMP(IL,I)=TI
      E1(IL,I)=B*Y/(QP*(1.-COS(Y*TI)))
      E2(IL,I)=LP*Y/(TI*(1.-SIN(Y*TI)/(Y*TI)))
   40 CONTINUE
      GRAFI=IQ*10.+IV*1.+0.1
      CALL  GRAFIC(GRAFI,NL,NT,OMEGAT,TIMP,IER)
   50 CONTINUE
      STOP
      END
```

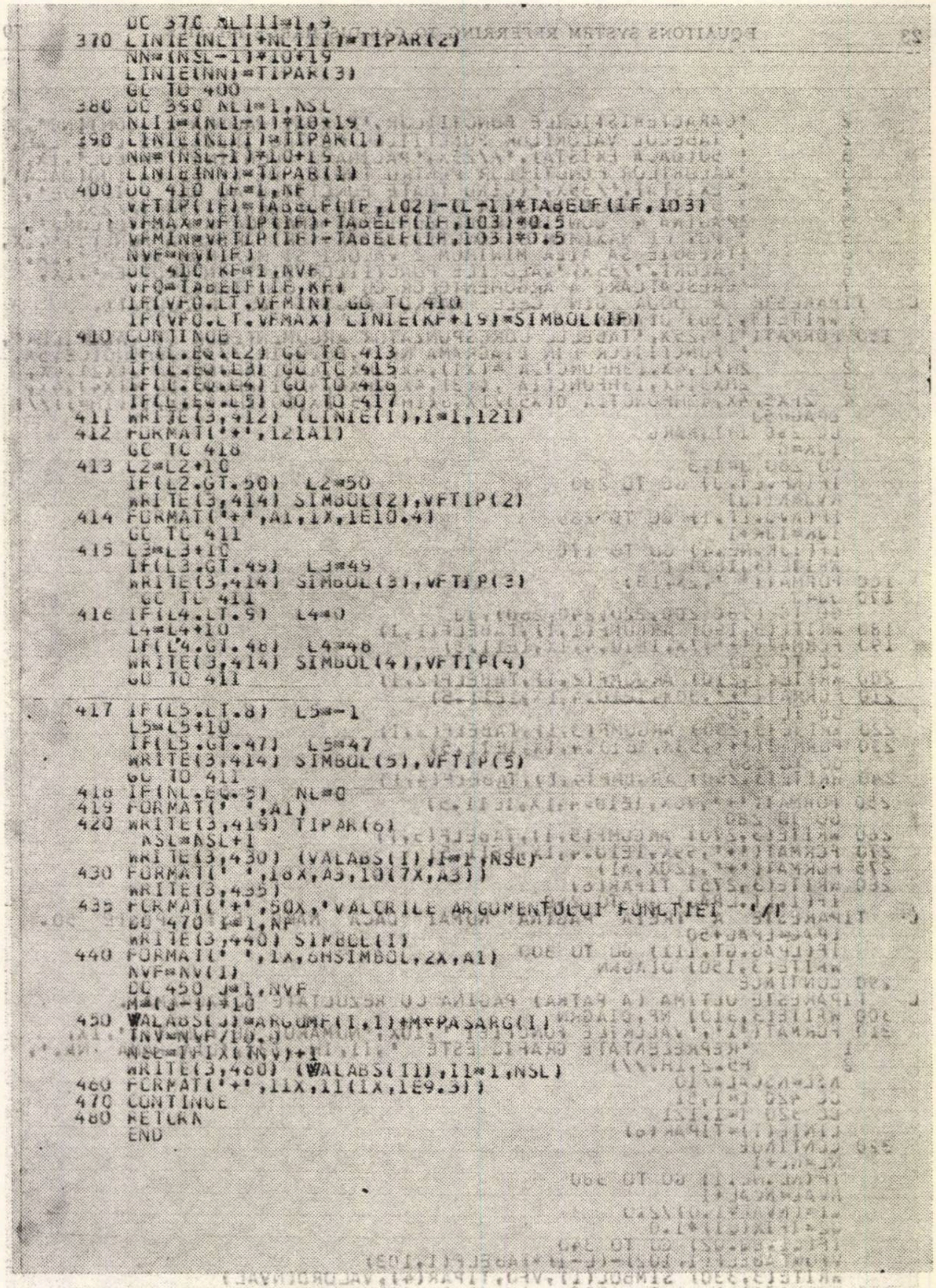
```
      SUBROUTINE  PUNCT(OMEGA,X1,XA,TOL,TI)
C
      DX=X1/10.0
      X2=X1+ABS(DX)
      DIF1=X1-SIN(X1)-XA*(1.0-COS(X1))
   10 DIF2=X2-SIN(X2)-XA*(1.0-COS(X2))
      IF(ABS(DIF2)-TOL) 90,90,20
   20 IF(DIF1*DIF2) 40,90,30
   30 DIF1=DIF2
      X1=X2
      X2=X2+DX
      GO TO 10
   40 X0=X1+(X2-X1)*ABS(DIF1)/(ABS(DIF1)+ABS(DIF2))
      DIF0=X0-SIN(X0)-XA*(1.0-COS(X0))
      IF(ABS(DIF0)-TOL) 80,80,50
   50 IF(DIF0*DIF1) 70,80,60
   60 X1=X0
      DIF1=DIF0
      GO TO 40
   70 X2=X0
      DIF2=DIF0
      GO TO 40
   80 TI=X0/OMEGA
      GO TO 100
   90 TI=X2/OMEGA
  100 RETURN
      END
```



```
      SUBROUTINE  GRAFIC(DIAGRN,NF,NV,ARG,TABELF,IER)
C
C  SUBRUTINA PENTRU REPREZENTARE GRAFICA A FUNCTIILOR DE O VARIABILA
C
      DIMENSION NV(5),ARG(5,2),TABELF(5,103),ARGUMF(5,100),PASARG(5),
     1             VFTIP(5),SIMBOL(5),TIPAR(6),VALORD(11),VALABS(11)
      REAL  LINIE(121),WALABS(11)
      DATA  NARG,NL,NVAL/2,0,0/,VALORD/3H1.0,3H0.9,3H0.8,3H0.7,3H0.6,3H0
     1.5,3H0.4,3H0.3,3H0.2,3H0.1,3H0.0/,VALABS/3H0  ,3H10 ,3H20 ,3H30 ,3
     2H40 ,3H50 ,3H60 ,3H70 ,3H80 ,3H90 ,3H100/,SIMBOL/1H*,1H.,1H,,1H+,1
     3H0/,TIPAR/1HI,1H-,1H+,1H=,1H.,1H / ,L2,L3,L4,L5/2,3,4,5/
      IF(NF.LT.1) GO TO 50
      IF(NF.GT.5) GO TO 60
      DO 20 I=1,NF
      NVF=NV(I)
      IF(NVF.LT.2) GO TO 70
      IF(NVF.GT.100) GO TO 80
      IF(NARG.LT.NVF) NARG=NVF
      ARGMIN=ARG(I,1)
      ARGMAX=ARG(I,2)
      IF(ARGMIN.GE.ARGMAX) GO TO 90
      DARGUM=(ARGMAX-ARGMIN)/((NVF-1)*1.0)
      PASARG(I)=DARGUM
      FMIN=TABELF(I,1)
      FMAX=FMIN
      ARGUMF(I,1)=ARGMIN
      DO 10 J=2,NVF
      ARGUMF(I,J)=ARGUMF(I,J-1)+DARGUM
      FIJ=TABELF(I,J)
      IF(FMIN.GT.FIJ) FMIN=FIJ
      IF(FMAX.LT.FIJ) FMAX=FIJ
   10 CONTINUE
      TABELF(I,101)=FMIN
      TABELF(I,102)=FMAX
      TABELF(I,103)=(FMAX-FMIN)/50.0
   20 CONTINUE
      DO 30 I=1,10
      NSCALA=I*10+1
      IF(NARG.LT.NSCALA) GO TO 40
   30 CONTINUE
C    S-AU  FACUT  INITIALIZARILE.  INCEPE  TIPARIREA .
   40 L2=2
      L3=3
      L4=4
      L5=5
      IER=1
      GO TO 100
   50 IER=2
      GO TO 480
   60 IER=3
      GO TO 480
   70 IER=4
      GO TO 480
   80 IER=5
      GO TO 480
   90 IER=6
      GO TO 480
C    TIPARESTE PRIMA DIN CELE PATRU PAGINI DE IMPRIMANTA CU REZULTATE.
  100 WRITE(3,110) DIAGRN
  110 FORMAT('1',17X,'FUNCTIILE  REPREZENTATE  GRAFIC  IN  DIAGRAMA',
     1       '  NR.',F5.2,2X,'AU  CARACTERISTICILE',2X,
     2    'URMATOARE  '/18X,90(1H=)//14X,98(1H-)/14X,1HI,4X,7HFUNCTIA,
     34X,1HI,10X,8HVALOAREA,9X,1HI,8X,10HARGUMENTUL,7X,1HI,4X,
     419HPASUL  DE  TABELARE,3X,1HI/14X,1HI,1X,12H(SIMBOLUL DE,2X,1HI,
     527(1H-),1HI,25(1H-),1HI,26(1H-),1HI/14X,1HI,1X,13HREPREZENTARE),
     61X,1HI,4X,6HMINIMA,3X,1HI,4X,6HMAXIMA,3X,1HI,4X,5HMINIM,3X,1HI,
     74X,5HMAXIM,3X,1HI,1X,10HPE ABSCISA,1X,1HI,1X,11HPE ORDONATA,1X,
     81HI/14X,1HI,15(1H=),1HI,13(1H=),1HI,13(1H=),1HI,12(1H=),1HI,
     912(1H=),1HI,12(1H=),1HI,13(1H=),1HI)
      DO 130 I=1,NF
      AR1=ARG(I,1)
      AR2=ARG(I,2)
      WRITE(3,120) SIMBOL(I),TABELF(I,101),TABELF(I,102),
     1          AR1,AR2,PASARG(I),TABELF(I,103)
  120 FORMAT(' ',13X,1HI,7X,A1,7X,1HI,2(1X,1E11.5,1X,1HI),
     1          3(1X,1E10.4,1X,1HI),1X,1E11.5,1X,1HI/14X,1HI,15(1H-),
     2          1HI,2(13(1H-),1HI),3(12(1H-),1HI),13(1H-),1HI)
  130 CONTINUE
      WRITE(3,140)
  140 FORMAT(' ',13X,98(1H=)////20X,12HOBSERVATII  /20X,12(1H=)//25X,
     1          'SINT IN TOTAL 4 PAGINI DE IMPRIMANTA PENTRU ACEASTA',1X,
     1          'REPREZENTARE GRAFICA  '//25X,'PAGINA 1 ARE',1X,
```



```
     2        'CARACTERISTICILE FUNCTIILOR.'//25X,'PAGINA 2  CONTINE',
     2        ' TABELUL VALORILOR FUNCTIILOR PENTRU INDICI DE LA 1 LA',
     3        ' 50(DACA EXISTA).'//25X,'PAGINA 3  CONTINE TABELUL',1X,
     3        'VALORILOR FUNCTIILOR PENTRU INDICI DE LA 51 LA 100(DACA',
     4        ' EXISTA).'/35X,'(CIND TOATE FUNCTIILE AU MAI PUTIN DE',
     4        ' 51 DE VALORI,PAGINA 3 LIPSESTE).'/36X,67(1H-)//25X,
     5        'PAGINA 4  CONTINE REPREZENTAREA GRAFICA A FUNCTIILOR.',
     5        ' POT FI MAXIMUM 5 FUNCTII.'/35X,'ORICARE DIN FUNCTII',1X,
     6        'TREBUIE SA AIBA MINIMUM 2 VALORI SI MAXIMUM 100 DE',1X,
     6        'VALORI.'/35X,'VALORILE FUNCTIILOR SINT IN ORDINEA',1X,
     7        'CRESCATOARE A ARGUMENTELOR CU PAS CONSTANT.')
C   TIPARESTE  A  DOUA  DIN  CELE  PATRU  PAGINI  CU  REZULTATE.
      WRITE(3,150) DIAGRN
  150 FORMAT('1',25X,'TABELUL CORESPUNZATOR ARGUMENTELOR SI VALORILOR',
     1        ' FUNCTIILOR DIN DIAGRAMA NR.',F5.2,1H.//1X,6HINDICE,5X,
     2        2HX1,4X,13HFUNCTIA *(X1),4X,2HX2,4X,13HFUNCTIA .(X2),4X,
     3        2HX3,4X,13HFUNCTIA ,(X3),4X,2HX4, 4X,13HFUNCTIA +(X4),4X,
     4  2HX5,4X,13HFUNCTIA O(X5)/1X,6(1H=),5(1X,10(1H=),1X,11(1H=))//)
      LPAG=50
      DO 290 I=1,NARG
      IJK=0
      DO 280 J=1,5
      IF(NF.LT.J) GO TO 280
      NVJ=NV(J)
      IF(NVJ.LT.I) GO TO 280
      IJK=IJK+1
      IF(IJK.NE.1) GO TO 170
      WRITE(3,160) I
  160 FORMAT(' ',2X,I3)
  170 JJ=J
      GO TO (180,200,220,240,260),JJ
  180 WRITE(3,190) ARGUMF(1,I),TABELF(1,I)
  190 FORMAT('+',7X,1E10.4,1X,1E11.5)
      GO TO 280
  200 WRITE(3,210) ARGUMF(2,I),TABELF(2,I)
  210 FORMAT('+',30X,1E10.4,1X,1E11.5)
      GO TO 280
  220 WRITE(3,230) ARGUMF(3,I),TABELF(3,I)
  230 FORMAT('+',53X,1E10.4,1X,1E11.5)
      GO TO 280
  240 WRITE(3,250) ARGUMF(4,I),TABELF(4,I)
  250 FORMAT('+',76X,1E10.4,1X,1E11.5)
      GO TO 280
  260 WRITE(3,270) ARGUMF(5,I),TABELF(5,I)
  270 FORMAT('+',99X,1E10.4,1X,1E11.5)
  275 FORMAT('+',120X,A1)
  280 WRITE(3,275) TIPAR(6)
      IF(I.LT.LPAG) GO TO 290
C   TIPARESTE  A  TREIA  PAGINA  NUMAI  DACA  NARG  ESTE  PESTE  50.
      LPAG=LPAG+50
      IF(LPAG.GT.111) GO TO 300
      WRITE(3,150) DIAGRN
  290 CONTINUE
C   TIPARESTE ULTIMA (A PATRA) PAGINA CU REZULTATE (DIAGRAMA).
  300 WRITE(3,310) NF,DIAGRN
  310 FORMAT('1','VALORILE FUNCTIEI ',10X,'NUMARUL DE FUNCTII',1X,
     1        'REPREZENTATE GRAFIC ESTE  ',I1,1H.,13X,'DIAGRAMA  NR.',
     2        F5.2,1H.//)
      NSL=NSCALA/10
      DO 420 L=1,51
      DO 320 I=1,121
      LINIE(I)=TIPAR(6)
  320 CONTINUE
      NL=NL+1
      IF(NL.NE.1) GO TO 380
      NVAL=NVAL+1
      Q1=(NVAL*1.0)/2.0
      Q2=IFIX(Q1)*1.0
      IF(Q1.EQ.Q2) GO TO 340
      VFO=TABELF(1,102)-(L-1)*TABELF(1,103)
      WRITE(3,330) SIMBOL(1),VFO,TIPAR(4),VALORD(NVAL)
  330 FORMAT('+',A1,1X,1E10.4,1X,A1,1X,A3)
      GO TO 360
  340 WRITE(3,350) VALORD(NVAL)
  350 FORMAT('+',15X,A3)
  360 DO 370 NLI=1,NSL
      NLII=(NLI-1)*10+19
      LINIE(NLII)=TIPAR(3)
```



```
      DO 370 NLIII=1,9
  370 LINIE(NLII+NLIII)=TIPAR(2)
      NN=(NSL-1)*10+19
      LINIE(NN)=TIPAR(3)
      GO TO 400
  380 DO 390 NLI=1,NSL
      NLII=(NLI-1)*10+19
  390 LINIE(NLII)=TIPAR(1)
      NN=(NSL-1)*10+19
      LINIE(NN)=TIPAR(1)
  400 DO 410 IF=1,NF
      VFTIP(IF)=TABELF(IF,102)-(L-1)*TABELF(IF,103)
      VFMAX=VFTIP(IF)+TABELF(IF,103)*0.5
      VFMIN=VFTIP(IF)-TABELF(IF,103)*0.5
      NVF=NV(IF)
      DO 410 KF=1,NVF
      VFO=TABELF(IF,KF)
      IF(VFO.LT.VFMIN) GO TO 410
      IF(VFO.LT.VFMAX) LINIE(KF+19)=SIMBOL(IF)
  410 CONTINUE
      IF(L.EQ.L2) GO TO 413
      IF(L.EQ.L3) GO TO 415
      IF(L.EQ.L4) GO TO 416
      IF(L.EQ.L5) GO TO 417
  411 WRITE(3,412) (LINIE(I),I=1,121)
  412 FORMAT('+',121A1)
      GO TO 418
  413 L2=L2+10
      IF(L2.GT.50)  L2=50
      WRITE(3,414) SIMBOL(2),VFTIP(2)
  414 FORMAT('+',A1,1X,1E10.4)
      GO TO 411
  415 L3=L3+10
      IF(L3.GT.49)  L3=49
      WRITE(3,414) SIMBOL(3),VFTIP(3)
       GO TO 411
  416 IF(L4.LT.9)  L4=0
      L4=L4+10
      IF(L4.GT.48)  L4=48
      WRITE(3,414) SIMBOL(4),VFTIP(4)
      GO TO 411
  417 IF(L5.LT.8)  L5=-1
      L5=L5+10
      IF(L5.GT.47)  L5=47
      WRITE(3,414) SIMBOL(5),VFTIP(5)
      GO TO 411
  418 IF(NL.EQ.5)  NL=0
  419 FORMAT(' ',A1)
  420 WRITE(3,419) TIPAR(6)
       NSL=NSL+1
      WRITE(3,430) (VALABS(I),I=1,NSL)
  430 FORMAT(' ',18X,A3,10(7X,A3))
      WRITE(3,435)
  435 FORMAT('+',50X,'VALORILE ARGUMENTULUI FUNCTIEI  '/)
      DO 470 I=1,NF
      WRITE(3,440) SIMBOL(I)
  440 FORMAT(' ',1X,6HSIMBOL,2X,A1)
      NVF=NV(I)
      DO 450 J=1,NVF
      M=(J-1)*10
  450 VALABS(J)=ARGUMF(I,1)+M*PASARG(I)
      INV=NVF/10.0
      NSL=IFIX(INV)+1
      WRITE(3,460) (VALABS(I1),I1=1,NSL)
  460 FORMAT('+',11X,11(1X,1E9.3))
  470 CONTINUE
  480 RETURN
      END
```

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